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STUDY OF AUTOMATIC CONTROL SYSTEMS FOR HELICOPTERS

Equations of Motion Including Rotor RPM Degree of Freedom for the Helicopter in Cruising Flight

31 August 1955

MINNEAPOLIS Honeywell

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STUDY OF AUTOMATIC CONTROL SYSTEMS

FOR HELICOPTERS

Equations of Motion Including

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M-H Aero Report AD 5143-TR13
Contract No. Nonr-929(00)

31 August 1955

Prepared by:

Paul Warsett
Paul Warsett
Research Project Engineer

Approved by:

Robert Maze
Robert Maze
Research Section Head

O Hugo Schuck
O. Hugo Schuck KAB
Director of Aero Research

Aeronautical Division
Minneapolis-Honeywell Regulator Company
Minneapolis, Minnesota

[redacted]

FOREWORD

This technical report was prepared by the Research Department, Aeronautical Division, Minneapolis-Honeywell Regulator Company, under Navy Contract No. Nonr-929(00), administered by the Office of Naval Research. This contract is sponsored jointly by the Air Branch, ONR, and the Power Plant Division, BuAer. This is a contract for research involving the study of helicopter control systems from the point of view of automatic control of flight attitude, altitude, and rotor rpm.

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ABSTRACT

This report contains the development of the theoretical equations defining the motions of a helicopter experiencing transient disturbances from steady-state cruising flight. The helicopter chosen for this study was of the single main rotor type, employing a tail rotor for torque compensation.

Included in this treatment of helicopter motion is the consideration of the influence of simultaneous transients in rotor RPM. In addition, decoupling of the longitudinal and lateral modes of motion was avoided.

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SECTION I

INTRODUCTION

1.1 INTRODUCTORY REMARKS

The research effort of which the presently reported work is a part deals with the subject of automatic, 'pilot-relief' control of helicopter flight attitude, altitude, and rotor RPM. Because of the interrelationship between these flight variables, optimum manual flight control can be achieved only by properly coordinated efforts of skilled pilots. Similarly, improved performance of automatic controls might be achieved through integrated action. This requires understanding of the dynamical relationships existing between the variables with which the several controls are simultaneously concerned. As in Ref. (1), which deals with hovering flight, these variables include pitching, rolling, and yawing of the helicopter, the variation in rotor RPM, and as auxiliary variables, the translatory disturbance motions of the helicopter and flapping disturbance motions of the rotor blades.

These considerations led to the requirement for a set of equations defining the helicopter transient motion, with due allowance for the possible effects of changes in rotor rpm as well as for the coupling phenomena between the longitudinal and lateral degrees of freedom. The development of these equations for hovering flight was described in Ref. (1), and the present report contains the development for the cruising regime.

1.2 ASSUMPTIONS MADE IN THE ANALYSIS

a) The equations were linearized by presuming all displacements from steady-state cruising conditions to be small.

b) It was presumed that no drag hinges were employed on the rotor.

c) The main rotor blades and rotor drive shaft were assumed to be infinitely stiff.

d) With regards to rotor blade motion, moments, etc., all harmonics above the first were taken to be negligibly small.

e) The slope of the lift curve of the blade elements was presumed to be constant.

1.3 THE COORDINATE SYSTEM AND FLIGHT GEOMETRY

The helicopter's motion is referred to coordinate axes which are moving in an unaccelerated manner in the direction and with the speed of the aircraft's C.G. in steady-state flight. If the axis system, with respect to which the motion is investigated, has a uniform translatory motion, the relative motion of the helicopter can be treated as if the axis system were at rest. Thus, Newton's laws can be applied to the motion with respect to these moving axes.

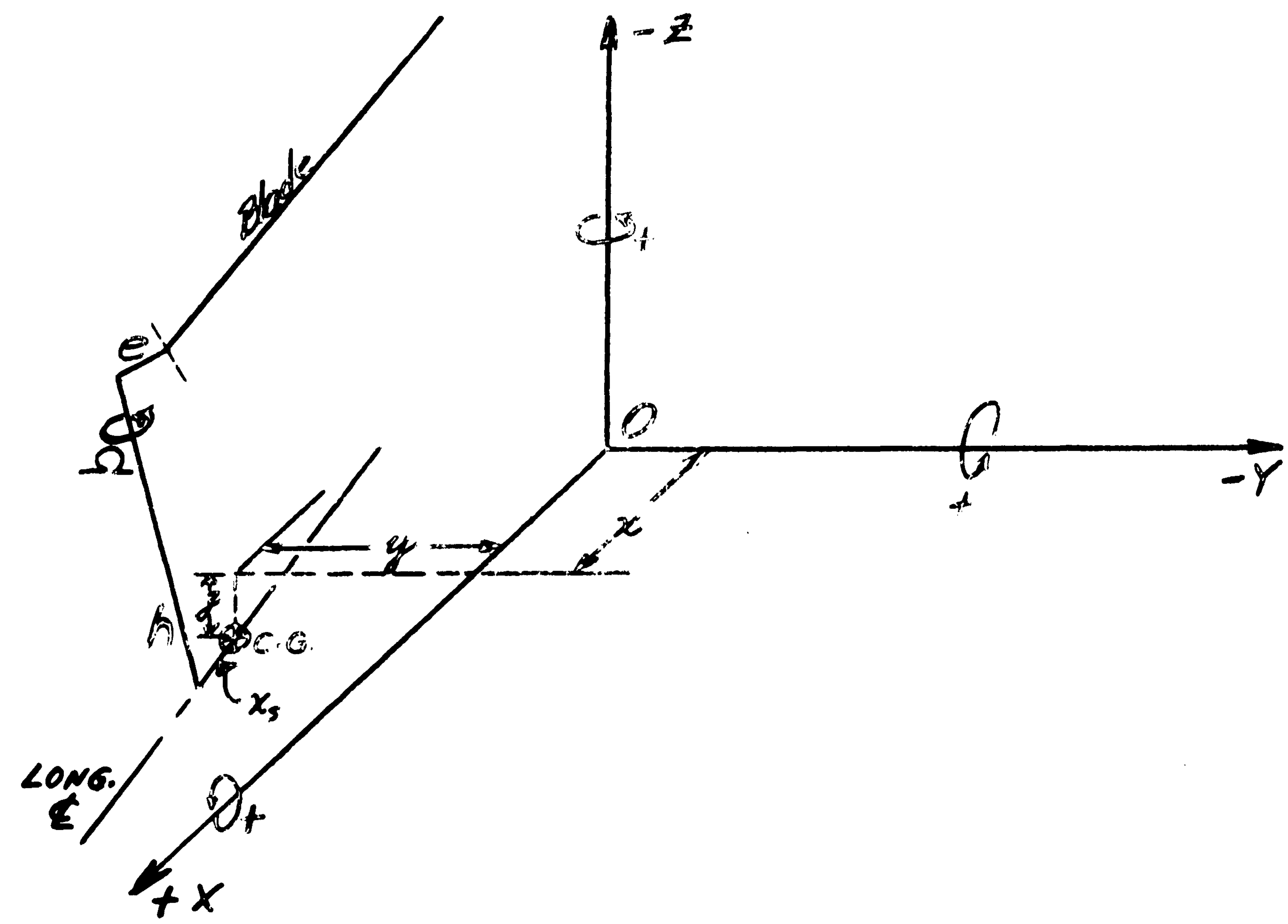

Figure 1

The Helicopter within the Moving Coordinate System

In Figure 1, the x, y, z location of the C.G. with respect to the center of coordinates O is illustrated. The point O is located at the position in space which would have been occupied by the C.G. in the absence of transient disturbances. It is indicated in Figure 1 that the helicopter C.G.* may not be located on the shaft axis. The directions

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* Throughout this development, the helicopter C.G. refers to the C.G. of the machine without main rotor blades.

of positive linear and angular displacements are shown in Figure 1 and in the two sketches following.

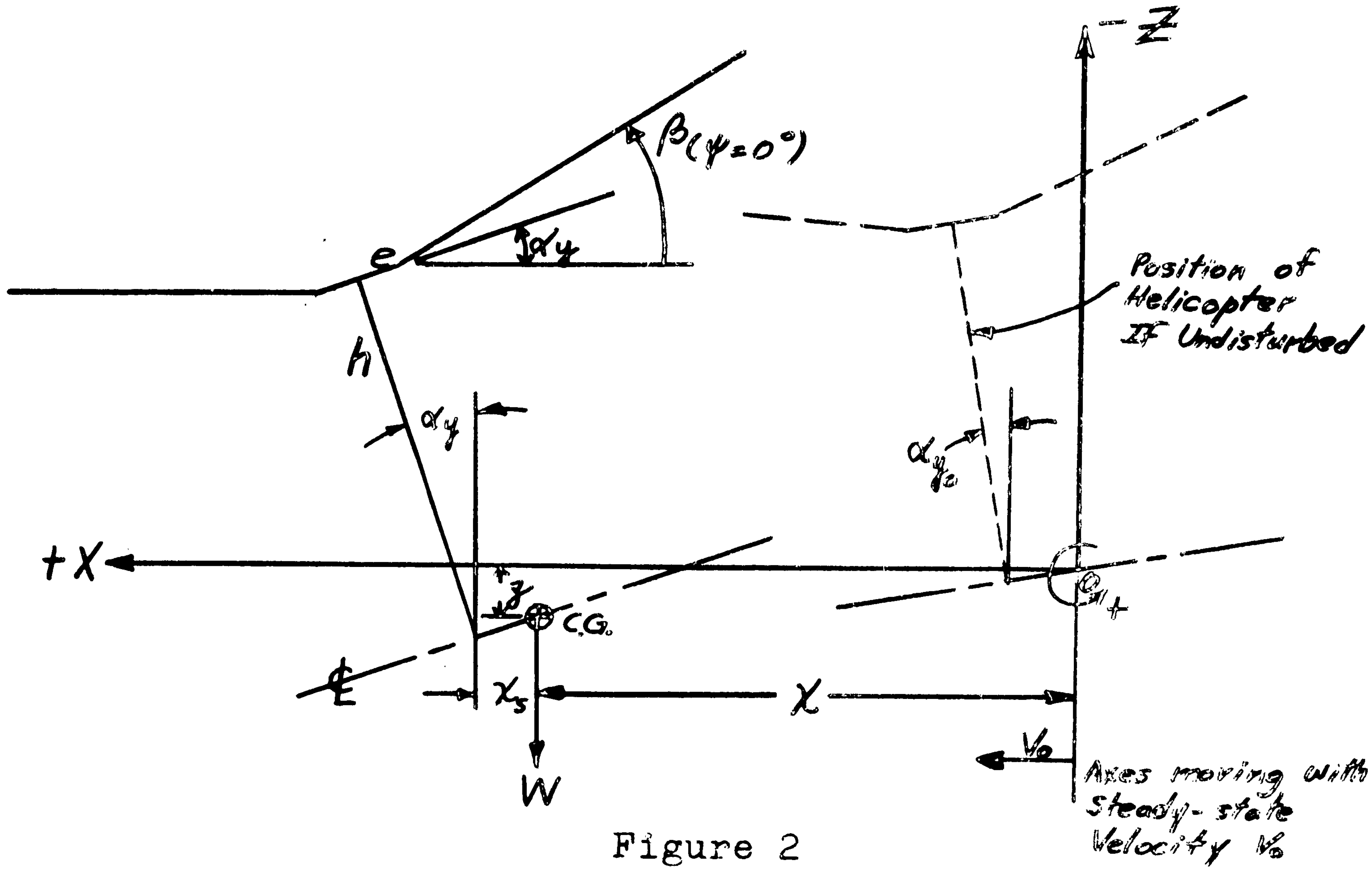

Figure 2

Helicopter Viewed in XZ Plane (ψ= 0°)

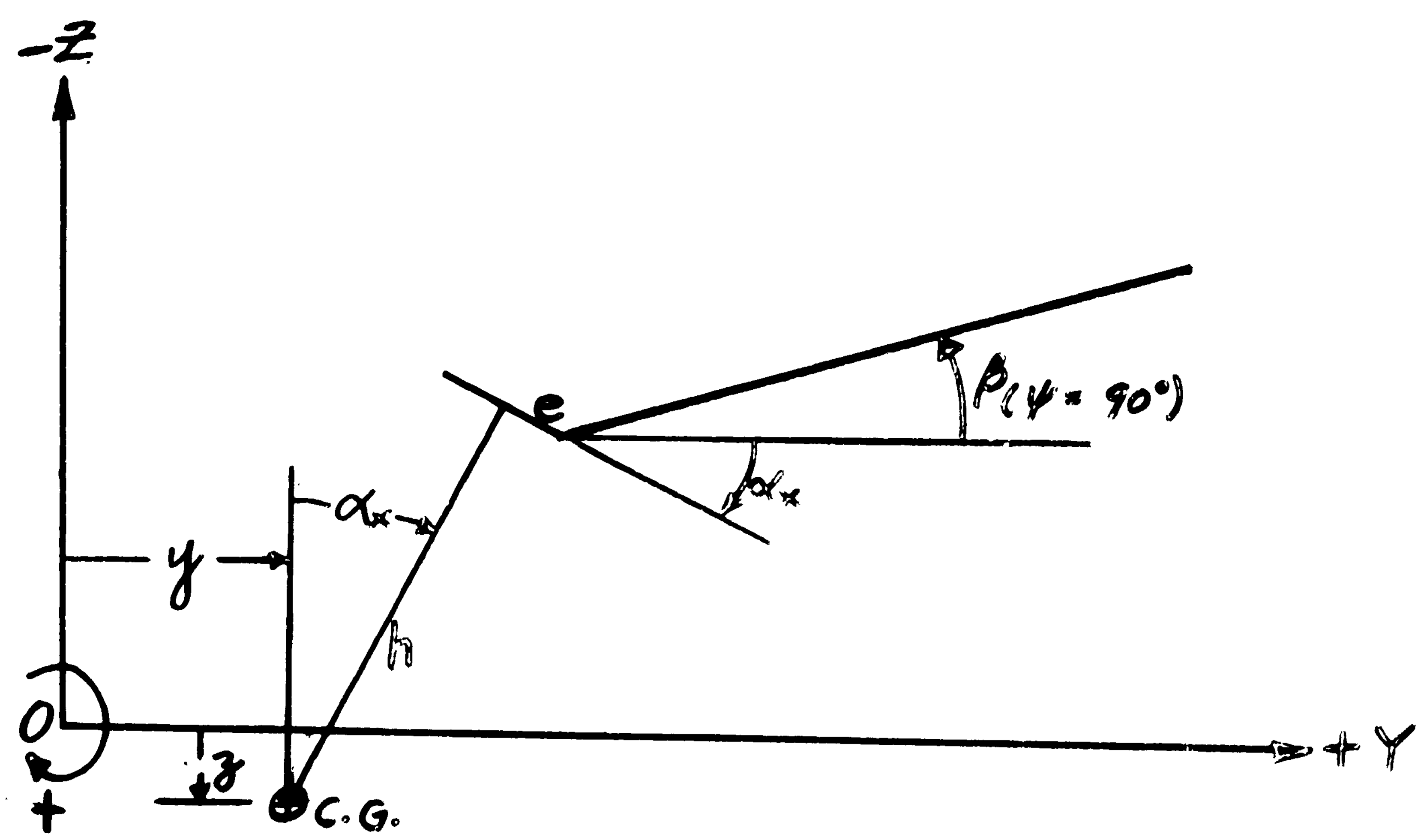

Figure 3

Helicopter Viewed in YZ Plane (ψ= 90°)

As indicated in Figures 2 and 3, the projection of the rotor shaft length h onto the XZ and YZ planes is presumed equal to h, since the angles α_x and α_y are taken to be small enough that

$$\cos \alpha_x = \cos \alpha_y = 1$$
$$\sin \alpha_x = \alpha_x \; ; \quad \sin \alpha_y = \alpha_y \tag{1}$$

Because we can assume that, even in high-speed cruising flight, the helicopter experiences relatively small angles of pitch and roll, angular displacements of the rotor blades in the plane normal to the rotor shaft are taken to be given by angular displacements measured in the XY plane.

SECTION II

EQUATIONS OF BLADE FLAPPING

2.1 BLADE ELEMENT KINEMATICS

It is assumed that the blade flapping motion and the basic forces can be determined on the basis of $e = 0$.

The x coordinate of a point on the blade span at radius r, as determined from Figure 2, is

$$x_b = x + x_s + h\alpha_y - r\cos\beta\cos\psi \tag{2}$$

The flapping angle β is given by

$$\beta = \beta_c + \beta_y\cos\psi + \beta_x\sin\psi \tag{3}$$

and is assumed to be small enough such that

$$\sin\beta = \beta\;;\quad \cos\beta = 1 \tag{4}$$

Writing the variables in terms of their steady-state and transient components, equation (2) becomes

$$\begin{aligned} x_b &= x + x_s + h(\alpha_{y_0} + \alpha_{y_\Delta}) - r\cos(\psi_0 + \psi_\Delta) \\ &= x + x_s + h(\alpha_{y_0} + \alpha_{y_\Delta}) - r(\cos\psi_0 - \psi_\Delta\sin\psi_0) \end{aligned} \tag{5}$$

Differentiating,

$$\dot{x}_b = \dot{x} + h\dot{\alpha}_{y_\Delta} + r\left[(\Omega_0 + \Omega_\Delta)\sin\psi_0 + \psi_\Delta\Omega_0\cos\psi_0\right] \tag{6}$$

$$\begin{aligned} \ddot{x}_b = \ddot{x} + h\ddot{\alpha}_{y_\Delta} + r\big[&(\dot{\Omega}_\Delta - \psi_\Delta\Omega_0^2)\sin\psi_0 \\ &+ (\Omega_0^2 + 2\Omega_\Delta\Omega_0)\cos\psi_0\big] \end{aligned} \tag{7}$$

Similarly,

$$y_b = y + h\,\alpha_{x_\Delta} + r(\sin\psi_0 + \psi_\Delta\cos\psi_0) \tag{8}$$

$$\dot{y}_b = \dot{y} + h\,\dot{\alpha}_{x_\Delta} + r\left[(\Omega_0 + \Omega_\Delta)\cos\psi_0 - \Omega_0\psi_\Delta\sin\psi_0\right] \tag{9}$$

$$\ddot{y}_b = \ddot{y} + h\ddot{\alpha}_{x_\Delta} + r\left[(\dot{\Omega}_\Delta - \psi_\Delta \Omega_0^2)\cos\psi_0 - (\Omega_0^2 + 2\Omega_0\Omega_\Delta)\sin\psi_0\right] \tag{10}$$

$$z_b = z - h - r\beta \tag{11}$$

From equation (3), written in steady-state and transient terms,

$$\beta = (\beta_{c_s} + \beta_{c_\Delta}) + (\beta_{y_s} + \beta_{y_\Delta})\cos(\psi_0 + \psi_\Delta) + (\beta_{x_s} + \beta_{x_\Delta})\sin(\psi_0 + \psi_\Delta)$$

or

$$\beta = (\beta_{c_s} + \beta_{c_\Delta}) + (\beta_{y_s} + \beta_{y_\Delta} + \beta_{x_s}\psi_\Delta)\cos\psi_0 + (\beta_{x_s} + \beta_{x_\Delta} - \beta_{y_s}\psi_\Delta)\sin\psi_0 \tag{12}$$

Hence, from equations (11) and (12),

$$\dot{z}_b = \dot{z} - r[\dot{\beta}_{c_\Delta} + (\dot{\beta}_{y_\Delta} + \beta_{x_s}\Omega_0 + \beta_{x_s}\Omega_\Delta + \beta_{x_\Delta}\Omega_0 - \beta_{y_s}\psi_\Delta\Omega_0)\cos\psi_0 + (\dot{\beta}_{x_\Delta} - \beta_{y_s}\Omega_0 - \beta_{y_s}\Omega_\Delta - \beta_{y_\Delta}\Omega_0 - \beta_{x_s}\psi_\Delta\Omega_0)\sin\psi_0] \tag{13}$$

$$\ddot{z}_b = \ddot{z} - r[\ddot{\beta}_{c_\Delta} + (\ddot{\beta}_{y_\Delta} + \beta_{x_s}\dot{\Omega}_\Delta + 2\Omega_0\dot{\beta}_{x_\Delta} - 2\beta_{y_s}\Omega_\Delta\Omega_0 - 2\beta_{y_s}\Omega_\Delta\Omega_0 - \beta_{y_s}\Omega_0^2 - \beta_{y_\Delta}\Omega_0^2 - \beta_{x_s}\psi_\Delta\Omega_0^2)\cos\psi_0 + (\ddot{\beta}_{x_\Delta} - \beta_{y_s}\dot{\Omega}_\Delta - 2\Omega_0\dot{\beta}_{y_\Delta} - 2\beta_{x_s}\Omega_\Delta\Omega_0 - \beta_{x_s}\Omega_0^2 - \beta_{x_\Delta}\Omega_0^2 + \beta_{y_s}\Omega_0^2\psi_\Delta)\sin\psi_0 \tag{14}$$

2.2 THE INERTIA MOMENT

The inertia force and the inertia moment about the flapping hinge can now be determined:

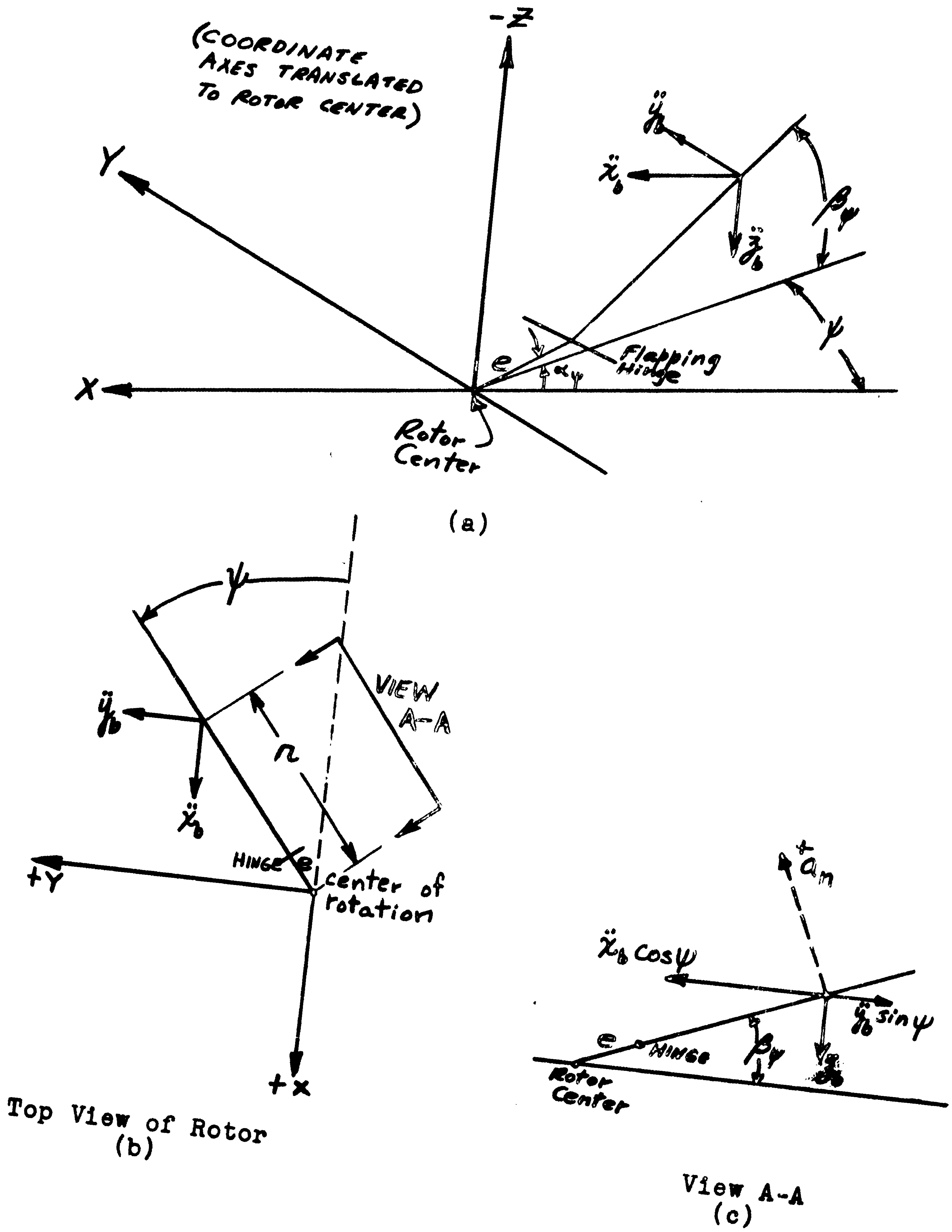

Figure 4

Blade Element Acceleration

Note that in Figure 4 the flapping hinge is taken to be parallel to the XY plane. While the flapping hinge will generally be inclined as shown, for example, in Figure 5 of Ref. (1), it was found (Ref. 1) that the moments about the flapping hinge could be adequately determined ignoring this complication. The normal acceleration (Figure 4-c) is given by

$$a_n = (\ddot{x}_b \cos\psi - \ddot{y}_b \sin\psi)\beta_\psi - \ddot{z}_b \tag{15}$$

Corresponding to the acceleration given by equation (15) is an elemental inertial force, $dF_i = a_n \, dm_b$, directed oppositely to a_n. This is the only inertia force component having a moment about the flapping hinge axis, given by

$$M_i = \int_0^R r\left[(\ddot{x}_b \cos\psi - \ddot{y}_b \sin\psi)\beta_\psi - \ddot{z}_b\right] dm_b \tag{16}$$

With

$$dm_b = \bar{\rho}\, dr \tag{17}$$

and retaining only first harmonic terms in ψ_0 , this leads to

$$\begin{aligned} M_i = \Big[& I_b\left[2\Omega_\Delta \Omega_0 \beta_{c_s} + \Omega_0^2(\beta_{c_s} + \beta_{c_\Delta}) + \ddot{\beta}_{c_\Delta}\right] \\ & + J_b\left[\frac{\beta_{y_s}}{2}(\ddot{x} + h\ddot{\alpha}_{y_\Delta}) - \frac{\beta_{x_s}}{2}(\ddot{y} + h\ddot{\alpha}_{x_\Delta}) - \ddot{z}\right]\Big] \\ & + \Big[I_b\left[\ddot{\beta}_{y_\Delta} + \beta_{x_s}\dot{\Omega}_\Delta + 2\Omega_0 \dot{\beta}_{x_\Delta}\right] \\ & \quad + J_b\left[(\ddot{x} + h\ddot{\alpha}_{y_\Delta})\beta_{c_s}\right]\Big]\cos\psi_0 \\ & + \Big[I_b\left[\ddot{\beta}_{x_\Delta} - \beta_{y_s}\dot{\Omega}_\Delta - 2\Omega_0 \dot{\beta}_{y_\Delta}\right] \\ & \quad - J_b\left[(\ddot{y} + h\ddot{\alpha}_{x_\Delta})\beta_{c_s}\right]\Big]\sin\psi_0 \end{aligned} \tag{18}$$

In equation (18), use has been made of the integral symbols, I_b and J_b , defined in the Appendix.

2.3 THE AERODYNAMIC MOMENT

Several assumptions were made in determining the moment about the flapping hinge due to air loads.

(a) The effect of the blade element drag on blade flapping was neglected.

(b) The wind angle ϕ (Figure 5) was taken to be small enough such that $\phi = U_P/U_T$.

(c) The induced velocity was presumed constant over the rotor disc.

(d) The lift-supporting portion of the blade span (of the HRS-3 helicopter) which is subjected to reverse flow in high-speed cruising flight is quite small. Special consideration of this problem was therefore eliminated from the development, and a single expression was employed for the aerodynamic moment of a blade element for all blade azimuth positions.

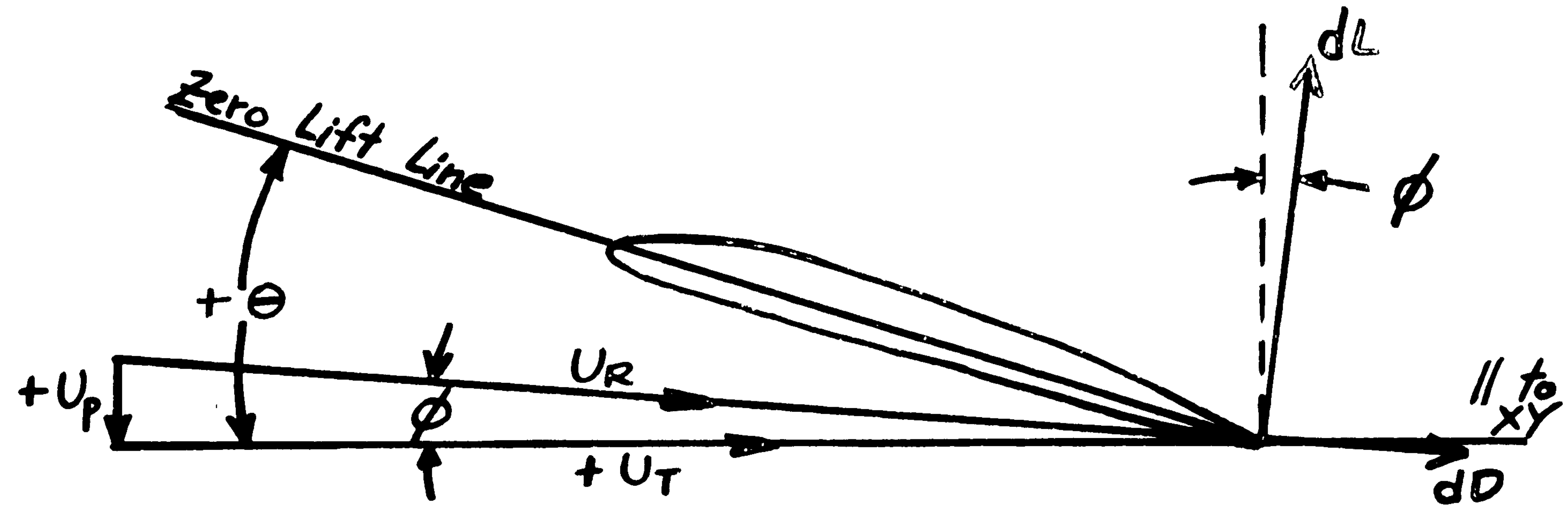

Figure 5

Conditions at the Blade Element

The elemental lift force is given by

$$\begin{aligned} dL &= C_L \frac{\rho}{2} U_R^2 c \, dr \\ &= a(\Theta - \phi) \frac{\rho}{2} U_T^2 c \, dr \\ &= \frac{\rho c a}{2} \left(\Theta U_T^2 - U_P U_T \right) dr \end{aligned} \tag{19}$$

Since U_T was chosen to be parallel to the XY plane,

$$U_T = \dot{x}_b \sin\psi + \dot{y}_b \cos\psi + V_0 \sin\psi \tag{20}$$

in which the last term accounts for the movement of the axis system. Expanding equation (20),

$$U_T = (\dot{x} + h\dot{\alpha}_{y_\Delta} + V_o)\sin\psi_o + r(\Omega_o + \Omega_\Delta) + (\dot{y} + h\dot{\alpha}_{x_\Delta} + V_o\psi_\Delta)\cos\psi_o \qquad (21)$$

The component velocity at the blade element normal to the XY plane, relative to the moving axis system, is

$$\dot{x}_b\cos\psi\sin\beta - \dot{y}_b\sin\psi\sin\beta - \dot{z}_b\cos\beta$$

To obtain the total velocity relative to still air (as required by equation 19), the inflow and steady-state velocity components

$$v + V_o\cos\psi_o\sin\beta$$

must be added, giving

$$U_P = \left[(\dot{x}_b + V_o)\cos\psi - \dot{y}_b\sin\psi\right]\beta + v - \dot{z}_b \qquad (22)$$

Expanding equation (22),

$$\begin{aligned} U_P = {} & \left[(\dot{x} + h\dot{\alpha}_{y_\Delta} + V_o)\cos\psi_o - (\dot{y} + h\dot{\alpha}_{x_\Delta} + V_o\psi_\Delta)\sin\psi_o\right] \cdot \\ & \cdot(\beta_{c_s} + \beta_{y_s}\cos\psi_o + \beta_{x_s}\sin\psi_o) + V_o\cos\psi_o\left[\beta_{c_\Delta}\right. \\ & \left. + (\beta_{y_\Delta} + \beta_{x_s}\psi_\Delta)\cos\psi_o + (\beta_{x_\Delta} - \beta_{y_s}\psi_\Delta)\sin\psi_o\right] \\ & + v - \dot{z} + r\left[\dot{\beta}_{c_\Delta} + (\dot{\beta}_{y_\Delta} + \beta_{x_s}\Omega_\Delta\right. \\ & + \beta_{x_s}\Omega_o + \beta_{x_\Delta}\Omega_o - \beta_{y_s}\Omega_o\psi_\Delta)\cos\psi_o \\ & + (\dot{\beta}_{x_\Delta} - \beta_{y_s}\Omega_\Delta - \beta_{y_s}\Omega_o - \beta_{y_\Delta}\Omega_o \\ & \left. - \beta_{x_s}\psi_\Delta\Omega_o)\sin\psi_o\right] \end{aligned} \qquad (23)$$

The only term in equation (19) remaining to be expressed is the pitch angle θ. This can be built up from the following considerations:

a) The pitch angle is composed of a "collective" part which does not vary with azimuth. This part may be expressed as the sum of a steady-state term and a transient (time-dependent) term, as

$$\theta = \left(\theta_{c_{s_{Root}}} - \frac{r}{R}\theta_d\right) + \theta_{c_\Delta}$$

The steady-state term provides for built-in blade twist. As written above, θ is measured from the blade element zero-lift-line and the plane normal to the rotor shaft.

b) In steady-state cruising flight, there is a cyclic variation in the pitch angle, of the form $(-\theta_{y_s}\sin\psi)$, also measured with respect to the plane normal to the rotor shaft. The minus sign corresponds to the sign convention assumed for pitch angle, as shown in Figure 5. Thus, in tilting the rotor disc downward in front, as required in forward flight, a negative blade pitch angle is required in the first quadrant of rotor revolution, as provided by the foregoing expression.

c) As indicated in Figure 5, the pitch angle must be obtained with respect to the horizontal plane XY. The angle between the XY plane and the plane normal to the rotor shaft, as measured in the airfoil plane of Figure 5, is

$$(-\alpha_y \sin\psi - \alpha_{x_\Delta}\cos\psi)$$

The minus signs are based on a presumption of positive pitching and rolling (Figure 1), coupled with the angular direction for positive blade element pitch angle in Figure 5. This expression can be best understood, perhaps, from a consideration of the blade at $\psi = 90°$ when viewing Figure 2, and at $\psi = 0°$ when viewing Figure 3.

d) During steady-state flight, it is presumed that the lateral cyclic pitch θ_x is zero. During the disturbance, provision must be made for transient changes in cyclic pitch, as follows:

$$(+\theta_{y_\Delta}\sin\psi + \theta_{x_\Delta}\cos\psi)$$

This assumes an increased nose-up blade pitch angle occurs in the first quadrant during the transient. This corresponds to the direction required to restore the helicopter to equilibrium attitude from the presumed disturbed position (pitch down, roll right - Figures 2 and 3).

On the basis of the above,

$$\theta = \left(\theta_{c_{s_{Root}}} - \frac{r}{R}\theta_d + \theta_{c_\Delta}\right) - \left(\theta_{y_s} - \theta_{y_\Delta} + \alpha_{y_0} + \alpha_{y_\Delta}\right)\sin\psi + \left(\theta_{x_\Delta} - \alpha_{x_\Delta}\right)\cos\psi$$

$$= \left(\theta_{c_{s_R}} - \frac{r}{R}\theta_d + \theta_{c_\Delta}\right) - \left(\theta_{y_s} - \theta_{y_\Delta} + \alpha_{y_0} + \alpha_{y_\Delta}\right)\sin\psi_0 + \left(\theta_{x_\Delta} - \alpha_{x_\Delta} - \theta_{y_s}\psi_\Delta - \alpha_{y_0}\psi_\Delta\right)\cos\psi_0 \tag{24}$$

The aerodynamic moment about the flapping hinge is given by

$$M_a = \int_0^{BR} r\,(dL) \tag{25}$$

which leads to

$$M_a = \theta_{c_{s_R}}\Big\{V_0(2\dot{x} + 2h\dot{\alpha}_{y_\Delta} + V_0)\frac{A_1}{2} + \Big[2\sin\psi_0[\Omega_0(\dot{x} + h\dot{\alpha}_{y_\Delta} + V_0) + \Omega_\Delta V_0] + 2\Omega_0\cos\psi_0(\dot{y} + h\dot{\alpha}_{x_\Delta} + V_0\psi_\Delta)\Big]C_1 + (\Omega_0^2 + 2\Omega_\Delta\Omega_0)B_1\Big\} - (\theta_{y_s} + \alpha_{y_0})\cdot$$
$$\cdot\Big\{\Big[V_0(2\dot{x} + 2h\dot{\alpha}_{y_\Delta} + V_0)\frac{3}{4}\sin\psi_0 + V_0(\dot{y} + h\dot{\alpha}_{x_\Delta} + V_0\psi_\Delta)\frac{1}{2}\cos\psi_0\Big]A_1 + [\Omega_0(\dot{x} + h\dot{\alpha}_{y_\Delta} + V_0) + \Omega_\Delta V_0]C_1 + \sin\psi_0(\Omega_0^2 + 2\Omega_\Delta\Omega_0)B_1\Big\}$$
$$- \frac{\theta_d}{R}\Big\{V_0(2\dot{x} + 2h\dot{\alpha}_{y_\Delta} + V_0)\frac{C_1}{2} + \Big[2\sin\psi_0[\Omega_0(\dot{x} + h\dot{\alpha}_{y_\Delta} + V_0) + \Omega_\Delta V_0] + 2\Omega_0\cos\psi_0(\dot{y} + h\dot{\alpha}_{x_\Delta} + V_0\psi_\Delta)\Big]B_1 + (\Omega_0^2 + 2\Omega_\Delta\Omega_0)D_1\Big\}$$
$$+ \theta_{c_\Delta}\left(V_0^2\frac{A_1}{2} + 2\Omega_0 V_0\sin\psi_0 C_1 + \Omega_0^2 B_1\right)$$

(CONT.)

$$
\begin{aligned}
&-(\alpha_{y_\Delta} - \theta_{y_\Delta})\big(V_0^2 \tfrac{3}{4} A_1 \sin\psi_0 + \Omega_0 V_0 C_1 \\
&+ \Omega_0^2 B_1 \sin\psi_0\big) + (\theta_{x_\Delta} - \alpha_{x_\Delta} - \psi_\Delta \theta_{y_s} - \psi_\Delta \alpha_{y_0}) \cdot \\
&\cdot \big(V_0^2 \tfrac{A_1}{4} \cos\psi_0 + \Omega_0^2 B_1 \cos\psi_0\big) - V_0 A_1 \Big\{ (\dot{x} \\
&+ h\dot{\alpha}_{y_\Delta} + V_0)\big(\tfrac{1}{4}\beta_{y_s} \sin\psi_0 + \tfrac{1}{4}\beta_{x_s}\cos\psi_0\big) - (\dot{y} \\
&+ h\dot{\alpha}_{x_\Delta} + V_0\psi_\Delta)\big(\tfrac{\beta_{c_s}}{2} + \tfrac{\beta_{y_s}}{4}\cos\psi_0 + \tfrac{3}{4}\beta_{x_s}\sin\psi_0\big) \\
&+ V_0\big[\tfrac{1}{4}(\beta_{y_\Delta} + \beta_{x_s}\psi_\Delta)\sin\psi_0 + \tfrac{1}{4}(\beta_{x_\Delta} - \beta_{y_s}\psi_\Delta)\cos\psi_0\big] \\
&+ v\sin\psi_0 - \dot{z}\sin\psi_0 \Big\} - \Omega_0 C_1 \Big\{ (\dot{x} + h\dot{\alpha}_{y_\Delta} + V_0) \cdot \\
&\cdot \big(\beta_{c_s}\cos\psi_0 + \tfrac{\beta_{y_s}}{2}\big) - (\dot{y} + h\dot{\alpha}_{x_\Delta} + V_0\psi_\Delta)\big(\beta_{c_s}\sin\psi_0 + \tfrac{\beta_{x_s}}{2}\big) \\
&+ V_0\big[\beta_{c_\Delta}\cos\psi_0 + \tfrac{1}{2}(\beta_{y_\Delta} + \beta_{x_s}\psi_\Delta)\big] + v - \dot{z} \Big\} - V_0 C_1 \big[\dot{\beta}_{c_\Delta}\sin\psi_0 + \tfrac{1}{2} \cdot \\
&\cdot (\dot{\beta}_{x_\Delta} - \beta_{y_s}\Omega_\Delta - \beta_{y_s}\Omega_0 - \beta_{y_\Delta}\Omega_0 - \beta_{x_s}\psi_\Delta\Omega_0)\big] - \Omega_0 B_1 \big[\dot{\beta}_{c_\Delta} \\
&+ \cos\psi_0 \,(\dot{\beta}_{y_\Delta} + \beta_{x_s}\Omega_\Delta + \beta_{x_s}\Omega_0 + \beta_{x_\Delta}\Omega_0 - \beta_{y_s}\Omega_0\psi_\Delta) \\
&+ \sin\psi_0 \,(\dot{\beta}_{x_\Delta} - \beta_{y_s}\Omega_\Delta - \beta_{y_s}\Omega_0 - \beta_{y_\Delta}\Omega_0 - \beta_{x_s}\psi_\Delta\Omega_0)\big] - (\dot{x} \\
&+ h\dot{\alpha}_{y_\Delta})A_1\big[V_0(\tfrac{1}{4}\beta_{y_s}\sin\psi_0 + \tfrac{1}{4}\beta_{x_s}\cos\psi_0) + v\sin\psi_0\big] - (\dot{y} \\
&+ h\dot{\alpha}_{x_\Delta} + V_0\psi_\Delta)A_1\big[V_0(\tfrac{1}{2}\beta_{c_s} + \tfrac{3}{4}\beta_{y_s}\cos\psi_0 + \tfrac{1}{4}\beta_{x_s}\sin\psi_0) \\
&+ v\cos\psi_0\big] - \Omega_\Delta C_1\big[V_0(\beta_{c_s}\cos\psi_0 + \tfrac{1}{2}\beta_{y_s}) + v\big] \\
&+ (\dot{x} + h\dot{\alpha}_{y_\Delta})C_1\,(\tfrac{1}{2}\beta_{y_s}\Omega_0) - (\dot{y} + h\dot{\alpha}_{x_\Delta} + V_0\psi_\Delta) \cdot \\
&\cdot C_1\,(\tfrac{1}{2}\beta_{x_s}\Omega_0) - \Omega_\Delta B_1\,(\beta_{x_s}\Omega_0\cos\psi_0 - \beta_{y_s}\Omega_0 \cdot \\
&\cdot \sin\psi_0)
\end{aligned}
\qquad (26)
$$

2.4 EQUATIONS OF BLADE MOTION

The equation for equilibrium of moments about the flapping hinge is

$$M_a - M_w - M_i = 0 \tag{27}$$

This equation must be satisfied at all times. Under these conditions, the coefficients of sin ψ_0, cos ψ_0, and unity must be separately identical to zero. Referring to equations (18) and (26), and omitting the steady-state terms (which form groups separately equal to zero anyway) including the blade weight moment, the following equations are developed from equation (27):

Unity Terms

$$\begin{aligned}
&\ddot{\beta}_{c_\Delta}(-I_b) + \dot{\beta}_{c_\Delta}(-\Omega_0 B_1) + \beta_{c_\Delta}(-I_b\Omega_0^2) + \Omega_\Delta \cdot \\
&\cdot\Big[-2\Omega_0\beta_{c_s} I_b + 2\theta_{c_{s_R}}\Omega_0 B_1 - (\theta_{y_s} + \alpha_{y_0})V_0 C_1 \\
&- 2\frac{\theta_d}{R}\Omega_0 D_1 - C_1 v\Big] + \ddot{z} J_b + \dot{z}\Omega_0 C_1 \\
&+ (\ddot{x} + h\ddot{\alpha}_{y_\Delta})\left(-\tfrac{1}{2}J_b\beta_{y_s}\right) + (\dot{x} + h\dot{\alpha}_{y_\Delta}) \cdot \\
&\cdot\Big[\theta_{c_{s_R}} V_0 A_1 - (\theta_{y_s} + \alpha_{y_0})\Omega_0 C_1 - \frac{\theta_d}{R} V_0 C_1\Big] \\
&+ (\ddot{y} + h\ddot{\alpha}_{x_\Delta})\left(\tfrac{1}{2}\beta_{x_s} J_b\right) + \dot{\beta}_{x_\Delta}\left(-\tfrac{1}{2}V_0 C_1\right) \\
&+ \theta_{c_\Delta}\left(\tfrac{1}{2}V_0^2 A_1 + \Omega_0^2 B_1\right) + (\alpha_{y_\Delta} - \theta_{y_\Delta})(-\Omega_0 V_0 C_1) = 0
\end{aligned} \tag{28}$$

Cosine Terms

$$\begin{aligned}
&\ddot{\beta}_{y_\Delta}(-I_b) + \dot{\beta}_{y_\Delta}(-\Omega_0 B_1) + \dot{\beta}_{x_\Delta}(-2\Omega_0 I_b) + \beta_{x_\Delta} \cdot \\
&\cdot\left(-\tfrac{1}{4}V_0^2 A_1 - \Omega_0^2 B_1\right) + \beta_{c_\Delta}(-\Omega_0 C_1 V_0) + \dot{\Omega}_\Delta(-\beta_{x_s} I_b) \\
&+ \Omega_\Delta(-\Omega_0 B_1\beta_{x_s} - C_1 V_0\beta_{c_s} - B_1\beta_{x_s}\Omega_0) + (\ddot{x} \\
&+ h\ddot{\alpha}_{y_\Delta})(-\beta_{c_s} J_b) + (\dot{x} + h\dot{\alpha}_{y_\Delta})\left(-\tfrac{1}{2}A_1 V_0\beta_{x_s}\right. \\
&\left.- \Omega_0 C_1\beta_{c_s}\right) + (\dot{y} + h\dot{\alpha}_{x_\Delta})\Big[2\theta_{c_{s_R}}\Omega_0 C_1
\end{aligned}$$

$$-(\theta_{y_s}+\alpha_{y_0})(\tfrac{1}{2}V_0A_1) - \frac{\theta_d}{R}\,2\Omega_0B_1 - \tfrac{1}{2}A_1V_0\beta_{y_s} - A_1v]$$
$$+\psi_\Delta\big[2\theta_{c_{s_R}}\Omega_0C_1V_0 - (\theta_{y_s}+\alpha_{y_0})\tfrac{1}{2}V_0^2A_1 - \frac{\theta_d}{R}\,2\Omega_0B_1V_0$$
$$-(\theta_{y_s}+\alpha_{y_0})(\tfrac{1}{4}V_0^2A_1+\Omega_0^2B_1)+\Omega_0^2B_1\beta_{y_s}-V_0A_1\,\cdot$$
$$\cdot\,(\tfrac{1}{4}V_0\beta_{y_s}+v)\big] + (\theta_{x_\Delta}-\alpha_{x_\Delta})(\tfrac{1}{4}V_0^2A_1+\Omega_0^2B_1)=0$$

(29)

Sine Terms

$$\ddot\beta_{x_\Delta}(-I_b)+\dot\beta_{x_\Delta}(-\Omega_0B_1)+\dot\beta_{y_\Delta}(2\Omega_0I_b)+\beta_{y_\Delta}(\Omega_0^2B_1$$
$$-\tfrac{1}{4}V_0^2A_1)+\dot\beta_{c_\Delta}(-V_0C_1)+\dot\Omega_\Delta(\beta_{y_s}I_b)+\Omega_\Delta\,\cdot$$
$$\cdot\,\big[2\theta_{c_{s_R}}C_1V_0-(\theta_{y_s}+\alpha_{y_0})(2B_1\Omega_0)-\frac{\theta_d}{R}(2B_1V_0)$$
$$+2\Omega_0B_1\beta_{y_s}\big]+\dot z V_0A_1+(\dot x+h\dot\alpha_{y_\Delta})\big[2\theta_{c_{s_R}}C_1\Omega_0$$
$$-(\theta_{y_s}+\alpha_{y_0})(\tfrac{3}{2}V_0A_1)-2\frac{\theta_d}{R}B_1\Omega_0-\tfrac{1}{2}A_1V_0\beta_{y_s}$$
$$-A_1v\big]+(\ddot y+h\ddot\alpha_{x_\Delta})J_b\beta_{c_s}+(\dot y+h\dot\alpha_{x_\Delta})\,\cdot$$
$$\cdot\,(\tfrac{1}{2}V_0A_1\beta_{x_s}+\Omega_0C_1\beta_{c_s})+\psi_\Delta(\Omega_0C_1V_0\beta_{c_s}$$
$$+\Omega_0^2B_1\beta_{x_s}+\tfrac{1}{4}V_0^2A_1\beta_{x_s})+(\alpha_{y_\Delta}-\theta_{y_\Delta})\,\cdot$$
$$\cdot\,(-\tfrac{3}{4}V_0^2A_1-\Omega_0^2B_1)+\theta_{c_\Delta}(2\Omega_0V_0C_1)=0$$

(30)

SECTION III

EQUATIONS OF HELICOPTER TRANSLATORY MOTION

3.1 THE BASIC EQUATIONS FOR TRANSLATION

If the force components acting at the helicopter C.G. in the X, Y, and Z directions are obtained, the translatory motion of the aircraft in these directions can be determined from the Newtonian relationships:

$$\sum F_X = F_{X_R} + F_{X_F} = \frac{W}{g}\ddot{x} \tag{31}$$

$$\sum F_Y = F_{Y_R} + T_{t_\Delta} = \frac{W}{g}\ddot{y} \tag{32}$$

$$\sum F_Z = F_{Z_R} = \frac{W}{g}\ddot{z} \tag{33}$$

Since all steady-state terms would cancel, the left-hand side of these equations should be written so that only transient terms appear. To determine the main-rotor force components, F_{X_R}, F_{Y_R}, and F_{Z_R}, acting on the free body helicopter, it is necessary to obtain the contributions from the individual blade elements on all b blades of the rotor.

3.2 THE MAIN ROTOR FORCES

The expressions for the blade element force components dF_x, dF_y, and dF_z can be taken directly from the corresponding development in hovering (Ref. 1). Thus, equation (51) in Ref. (1) gave

$$dF_{X_R} = -\ddot{x}_b\, dm_b + (dL)\cos\phi\sin\beta\cos\psi - (dL)\sin\phi\sin\psi - (dD)\cos\phi\sin\psi - (dD)\sin\phi\sin\beta\cos\psi \tag{34}$$

From equation (61) of Ref. (1),

$$dF_{Y_R} = -\ddot{y}_b\, dm_b - (dL)\cos\phi\sin\beta\sin\psi - (dL)\sin\phi\cos\psi - (dD)\cos\phi\cos\psi + (dD)\sin\phi\sin\beta\sin\psi \tag{35}$$

From equation (63) of Ref. (1),

$$dF_{z_R} = dW_b - \ddot{z}_b \, dm_b - (dL)\cos\phi \cos\beta + (dD)\sin\phi \cos\beta \tag{36}$$

Note that in equations (34), (35), and (36), it is possible to combine the last term in each case with a corresponding "lift" term; for example, in equation (34) this is

$$\sin\beta \cos\psi \left[dL \cos\phi - dD \sin\phi \right]$$

or

$$\sin\beta \cos\psi \left[dL - dD \frac{U_P}{U_T} \right]$$

or

$$(\sin\beta \cos\psi) \frac{\rho c a}{2} \left[(\theta - \phi) - \frac{C_{D_0}}{a} \frac{U_P}{U_T} \right] U_T^2 \, dr$$

or

$$(\sin\beta \cos\psi) \frac{\rho c a}{2} \left[\theta U_T^2 - U_P U_T \left(1 + \frac{C_{D_0}}{a}\right) \right] dr$$

Now, C_{D_0}/a is very small compared with 1, and can be neglected with only minor error. On this basis, the last term in each of the three equations (34-36) has been neglected.

The evaluation and integration of the three force equations (34-36) has been carried out in a manner similar to that indicated in Ref. (1), to yield:

$$\begin{aligned}
F_{x_R} = & -(\ddot{x} + h\ddot{\alpha}_{y_\Delta}) b \frac{W_b}{g} - (\dot{x} + h\dot{\alpha}_{y_\Delta}) \Big[\beta_{c_s}^2 \Omega_0 \frac{A_2}{2} \\
& + \beta_{y_s}^2 \Omega_0 \frac{A_2}{2} + \beta_{y_s} \Omega_0 \frac{A_2}{2} (\theta_{y_s} + \alpha_{y_0}) + v \cdot \\
& \cdot \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2} \right) + \Omega_0 A_3 \Big] + (\dot{y} + h\dot{\alpha}_{x_\Delta}) \cdot \\
& \cdot \Big[\beta_{c_s} \beta_{y_s} V_0 E_2 - (\theta_{y_s} + \alpha_{y_0}) \left(\beta_{c_s} V_0 \frac{E_2}{2} + \beta_{x_s} \Omega_0 \frac{A_2}{2} \right) \\
& + \beta_{x_s} \beta_{y_s} \Omega_0 \frac{A_2}{2} - \beta_{c_s} v \frac{3}{2} E_2 + \beta_{c_s} \Omega_0 \Big(\theta_{c_{s_R}} \frac{3A_2}{2} \\
& - \frac{\theta_d}{R} \frac{3C_2}{2} \Big) + \beta_{x_s} V_0 \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2} \right) \Big] \\
& + (\theta_{x_\Delta} - \alpha_{x_\Delta}) \left[\beta_{c_s} \Omega_0^2 \frac{C_2}{2} \right] - (\alpha_{y_\Delta} - \theta_{y_\Delta}) \cdot
\end{aligned}$$

$$
\cdot \left[\Omega_o V_o \beta_{y_s} \frac{A_2}{2} - v \Omega_o \frac{A_2}{2} \right] + \beta_{x_\Delta} \left[-\beta_{c_s} \Omega_o^2 \frac{C_2}{2} \right]
$$

$$
+ \beta_{y_\Delta} \left[-\beta_{y_s} V_o \Omega_o A_2 + \Omega_o^2 \left(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2 \right) - V_o \Omega_o \frac{A_2}{2} (\theta_{y_s}
$$

$$
+ \alpha_{y_o}) - \Omega_o v \frac{3A_2}{2} \right] + \dot{\beta}_{x_\Delta} \left[\beta_{y_s} V_o \frac{A_2}{8} - \Omega_o \left(\theta_{c_{s_R}} \frac{C_2}{2} - \frac{\theta_d}{R} \frac{B_2}{2} \right) \right.
$$

$$
\left. + V_o \frac{3A_2}{8} (\theta_{y_s} + \alpha_{y_o}) + v A_2 \right] + \dot{\beta}_{y_\Delta} \left[-\beta_{c_s} \Omega_o \frac{C_2}{2} + \beta_{x_s} V_o \frac{A_2}{8} \right]
$$

$$
+ \beta_{c_\Delta} \left[-\beta_{c_s} V_o \Omega_o A_2 - \beta_{x_s} \Omega_o^2 \frac{C_2}{2} \right] + \dot{\beta}_{c_\Delta} \left[-\beta_{y_s} \Omega_o \frac{3C_2}{2} \right.
$$

$$
\left. - V_o \left(\theta_{c_{s_R}} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2} \right) + \Omega_o \frac{C_2}{2} (\theta_{y_s} + \alpha_{y_o}) \right]
$$

$$
+ \Omega_\Delta \left[-\beta_{c_s} \beta_{x_s} \Omega_o C_2 + \beta_{y_s} \Omega_o \left(\theta_{c_{s_R}} 2 C_2 - \frac{\theta_d}{R} 2 B_2 \right) \right.
$$

$$
- (\theta_{y_s} + \alpha_{y_o}) \left(\beta_{y_s} V_o \frac{A_2}{2} - v \frac{A_2}{2} \right) - \beta_{c_s}^2 V_o \frac{A_2}{2} - \beta_{y_s}^2 V_o \frac{A_2}{2}
$$

$$
\left. - \beta_{y_s} v \frac{3}{2} A_2 - V_o A_3 \right] + \theta_{c_\Delta} \left[\beta_{y_s} \Omega_o^2 C_2 - V_o v \frac{E_2}{2} \right]
$$

$$
+ \dot{z} \left[V_o \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2} \right) - \Omega_o \frac{A_2}{2} (\theta_{y_s} + \alpha_{y_o}) + \beta_{y_s} \Omega_o \frac{3}{2} A_2 \right]
$$

$$
+ \left[-\beta_{c_s}^2 V_o \Omega_o \frac{A_2}{2} - \beta_{c_s} \beta_{x_s} \Omega_o^2 \frac{C_2}{2} + \beta_{y_s} \Omega_o^2 \left(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2 \right) \right.
$$

$$
- \beta_{y_s} \Omega_o V_o \frac{A_2}{2} (\theta_{y_s} + \alpha_{y_o}) - \beta_{y_s}^2 V_o \Omega_o \frac{A_2}{2} - \beta_{y_s} \Omega_o v \frac{3A_2}{2}
$$

$$
\left. - V_o v \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2} \right) + v \Omega_o \frac{A_2}{2} (\theta_{y_s} + \alpha_{y_o}) - \Omega_o V_o A_3 \right] \tag{37}
$$

$$
F_{Y_R} = -(\ddot{y} + h \ddot{\alpha}_{x_\Delta}) b \frac{W_b}{g} + (\dot{y} + h \dot{\alpha}_{x_\Delta}) \left[-\beta_{c_s} \beta_{x_s} V_o E_2 - \beta_{c_s}^2 \Omega_o \frac{A_2}{2} \right.
$$

$$
- \beta_{x_s}^2 \Omega_o \frac{A_2}{2} - \beta_{y_s} V_o \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2} \right) - v \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2} \right)
$$

$$
\left. - \Omega_o A_3 \right] + (\dot{x} + h \dot{\alpha}_{y_\Delta}) \left[(\theta_{y_s} + \alpha_{y_o}) (\beta_{c_s} V_o E_2 + \beta_{x_s} \Omega_o A_2) \right.
$$

$$
- 3 \beta_{c_s} \Omega_o \left(\theta_{c_{s_R}} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2} \right) - \beta_{x_s} V_o \left(\theta_{c_{s_R}} E_2 - \frac{\theta_d}{R} A_2 \right)
$$

$$
\left. + \beta_{c_s} \beta_{y_s} V_o 2 E_2 + \beta_{x_s} \beta_{y_s} \Omega_o \frac{A_2}{2} + \beta_{c_s} v \frac{3}{2} E_2 \right]
$$

$$
+ (\alpha_{y_\Delta} - \theta_{y_\Delta}) \left[\beta_{c_s} \left(V_o^2 \frac{E_2}{2} + \Omega_o^2 \frac{C_2}{2} \right) - \beta_{x_s} V_o \Omega_o A_2 \right]
$$

$$
+ (\theta_{x_\Delta} - \alpha_{x_\Delta}) \left[-\beta_{y_s} V_o \Omega_o \frac{A_2}{2} - \Omega_o v \frac{A_2}{2} \right] + \Omega_\Delta \left[(\theta_{y_s} + \right.
$$

$$
\begin{aligned}
&+ \alpha_{y_0})(\beta_{c_s}\Omega_0 C_2 + \beta_{x_s}V_0 A_2) - 3\beta_{c_s}V_0\left(\theta_{c_{s_R}}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right) - 2\beta_{x_s}\Omega_0\left(\theta_{c_{s_R}}C_2\right.\\
&\left.- \frac{\theta_d}{R}B_2\right) - \beta_{c_s}\beta_{y_s}\Omega_0 C_2 + \beta_{x_s}\left(v\frac{3}{2}A_2 + \beta_{y_s}V_0\frac{A_2}{2}\right)\Big] + \theta_{c_\Delta}\Big[-\beta_{x_s}\Big(V_0^2\frac{E_2}{2}\\
&+ \Omega_0^2 C_2\Big) - 3\beta_{c_s}V_0\Omega_0\frac{A_2}{2}\Big] + \dot\beta_{y_\Delta}\Big[\beta_{y_s}V_0\frac{7A_2}{8} - \Omega_0\left(\theta_{c_{s_R}}\frac{C_2}{2} - \frac{\theta_d}{R}\frac{B_2}{2}\right)\\
&+ vA_2 + V_0\frac{A_2}{8}(\theta_{y_s} + \alpha_{y_0})\Big] + \beta_{y_\Delta}\Big[\beta_{c_s}V_0^2E_2 - \beta_{c_s}\Omega_0^2\frac{C_2}{2}\\
&+ \beta_{x_s}V_0\Omega_0\frac{A_2}{2}\Big] + \dot z\Big[-\beta_{c_s}V_0\frac{3E_2}{2} - \beta_{x_s}\Omega_0\frac{3}{2}A_2\Big] + \dot\beta_{c_\Delta}\,\cdot\\
&\cdot\Big[\beta_{c_s}V_0\frac{3}{2}A_2 + \beta_{x_s}\Omega_0\frac{3}{2}C_2\Big] + \dot\beta_{x_\Delta}\Big[\beta_{c_s}\Omega_0\frac{C_2}{2} + \beta_{x_s}V_0\frac{5}{8}A_2\Big]\\
&+ \beta_{c_\Delta}\Big[(\theta_{y_s} + \alpha_{y_0})\left(V_0^2\frac{E_2}{2} + \Omega_0^2\frac{C_2}{2}\right) - 3V_0\Omega_0\Big(\theta_{c_{s_R}}\frac{A_2}{2}\\
&- \frac{\theta_d}{R}\frac{C_2}{2}\Big) + \beta_{y_s}V_0^2E_2 + V_0 v\frac{3E_2}{2} - \beta_{y_s}\Omega_0^2\frac{C_2}{2}\Big] + \beta_{x_\Delta}\,\cdot\\
&\cdot\Big[-V_0^2\left(\theta_{c_{s_R}}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) - \Omega_0^2\left(\theta_{c_{s_R}}C_2 - \frac{\theta_d}{R}B_2\right)\\
&+ V_0\Omega_0A_2(\theta_{y_s} + \alpha_{y_0}) + \beta_{y_s}V_0\Omega_0\frac{A_2}{2} + v\Omega_0\frac{3}{2}A_2\Big] + \Big[\beta_{c_s}\beta_{y_s}V_0^2E_2\\
&+ V_0 v\beta_{c_s}\frac{3}{2}E_2 - \beta_{c_s}\beta_{y_s}\Omega_0^2\frac{C_2}{2} + \beta_{c_s}V_0^2\frac{E_2}{2}(\theta_{y_s} + \alpha_{y_0})\\
&+ \beta_{c_s}\Omega_0^2\frac{C_2}{2}(\theta_{y_s} + \alpha_{y_0}) + \beta_{x_s}\beta_{y_s}V_0\Omega_0\frac{A_2}{2} + \beta_{x_s}v\Omega_0\,\cdot\\
&\cdot\frac{3}{2}A_2 - \beta_{x_s}V_0^2\left(\theta_{c_{s_R}}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) - \beta_{x_s}\Omega_0^2\left(\theta_{c_{s_R}}C_2 - \frac{\theta_d}{R}B_2\right)\\
&+ \beta_{x_s}\Omega_0V_0A_2(\theta_{y_s} + \alpha_{y_0}) - 3\beta_{c_s}V_0\Omega_0\left(\theta_{c_{s_R}}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right)\Big]
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
F_{z_R} = &-\ddot z b\frac{W_b}{g} + \ddot\beta_{c_\Delta}bJ_b + (\dot x + h\dot\alpha_{y_\Delta})\Big[-V_0\left(\theta_{c_{s_R}}E_2 - \frac{\theta_d}{R}A_2\right)\\
&+ (\theta_{y_s} + \alpha_{y_0})\Omega_0A_2\Big] - (\theta_{y_\Delta} - \alpha_{y_\Delta})\Big[\Omega_0V_0A_2\Big] + \Omega_\Delta\,\cdot\\
&\cdot\Big[-2\Omega_0\left(\theta_{c_{s_R}}C_2 - \frac{\theta_d}{R}B_2\right) + (\theta_{y_s} + \alpha_{y_0})V_0A_2 + vA_2\Big]
\end{aligned}
$$

$$+ \theta_{c_\Delta}\left[-V_0^2 \frac{\epsilon_z}{2} - \Omega_0^2 C_z\right] + \dot{z}\left(-\Omega_0 A_z\right) + \dot{\beta}_{c_\Delta}\left(\Omega_0 C_z\right)$$
$$+ \dot{\beta}_{x_\Delta}\left(V_0 \frac{A_z}{2}\right) + \left[b W_b - V_0^2\left(\theta_{c_{s_R}} \frac{\epsilon_z}{2} - \frac{\theta_d}{R}\frac{A_z}{2}\right)\right.$$
$$- \Omega_0^2\left(\theta_{c_{s_R}} C_z - \frac{\theta_d}{R} B_z\right) + \left(\theta_{y_s} + \alpha_{y_o}\right)\Omega_0 V_0 A_z$$
$$\left. + v\Omega_0 A_z\right] \tag{39}$$

3.3 THE FUSELAGE FORCE

During the transient disturbance, the helicopter fuselage may experience changes in fuselage drag and moments which contribute to the stability problem. An extensive review of existing, available information has been made to guide the present development in this matter. On this basis, it was assumed that the fuselage contributes a single force component, acting at the helicopter C.G. in the X direction, in accordance with

$$F_{x_F} = - \frac{\partial D_f}{\partial \dot{x}} \dot{x} \tag{40}$$

Now,

$$D_f = C_{D_f} \frac{\rho V^2}{2} \pi R^2$$
$$= C_{D_f} \frac{\rho}{2} \pi R^2 (V_0 + \dot{x})^2$$
$$= C_{D_f} \frac{\rho}{2} \pi R^2 (V_0^2 + 2V_0 \dot{x})$$

Thus,

$$\frac{\partial D_f}{\partial \dot{x}} = C_{D_f} \rho \pi R^2 V_0$$

Hence,

$$F_{x_F} = - \left(C_{D_f} \rho \pi R^2 V_0\right) \dot{x} \tag{41}$$

The variation of this force with changes in fuselage angular attitude has been ignored.

3.4 THE TAIL ROTOR FORCE

The following illustrations serve to define the conditions presumed to exist at the tail rotor.

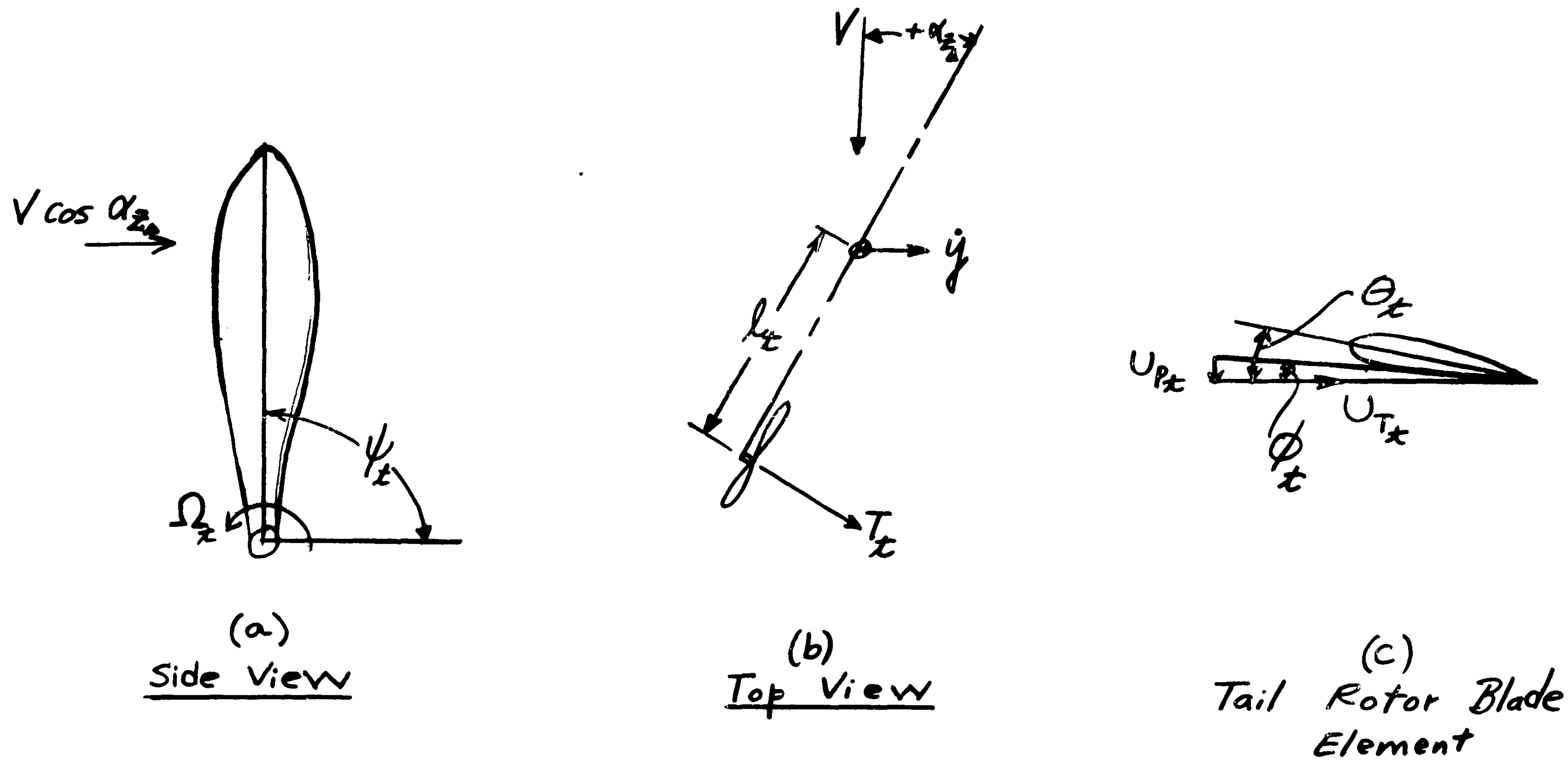

Figure 6

Tail Rotor

In this treatment, blade flapping is ignored, the inflow is presumed to be uniform, and the influence of the reverse flow region is ignored. With α_{z_Δ} a small angle,

$$U_{T_t} = \Omega_t r_t + V \sin \psi_t$$

$$= (\Omega_{o_t} + \Omega_{\Delta_t}) r_t + V(\sin \psi_{o_t} + \psi_{\Delta_t} \cos \psi_{o_t}) \quad (42)$$

$$U_{P_t} = v_t - l_t \dot{\alpha}_{z_\Delta} + \dot{y} - V \cdot \alpha_{z_\Delta} \quad (43)$$

$$\theta_t = \theta_{o_t} + \theta_{\Delta_t} \quad (44)$$

$$r_t = C_T + \frac{R_t - r_t}{R_t}\left(C_R - C_T\right) \tag{45}$$

$$dT_t = dL_t = \frac{\rho c_t a_t}{2}\left(\theta_t U_{T_t}^2 - U_{P_t} U_{T_t}\right) dr_t \tag{46}$$

With $V = V_0 + \dot{x}$, equation (46) gives, for b_t blades,

$$\begin{aligned} T_t = {} & (\theta_{0_t} + \theta_{\Delta_t})\left[\Omega_{0_t}^2 C_4 + 2\Omega_{0_t}\Omega_{\Delta_t} C_4 \right. \\ & \left. + V_0^2 E_4/2 + V_0 \dot{x} E_4\right] - \left[\Omega_{0_t}\left(v_t \right.\right. \\ & \left.\left. - l_t \dot{\alpha}_{z_\Delta} + \dot{y} - V_0 \alpha_{z_\Delta}\right) A_4 + \Omega_{\Delta_t} v_t A_4\right] \end{aligned} \tag{47}$$

3.5 THE EQUATIONS OF TRANSLATORY MOTION

It is now only a matter of substituting into the relationships of equations (31-33) to obtain the equations of translatory motion. The required substitutions are obtained from equations (37), (38), (39), (41), and (47). In terms of the stability derivatives as coefficients, the following three equations are derived:

$$\begin{aligned} & X_{\ddot{x}}\ddot{x} + X_{\dot{x}}\dot{x} + X_{\dot{y}}\dot{y} + X_{\ddot{\alpha}_{y_\Delta}}\ddot{\alpha}_{y_\Delta} + X_{\dot{\alpha}_y}\dot{\alpha}_y \\ & + X_{\alpha_{y_\Delta}}\alpha_{y_\Delta} + X_{\dot{\alpha}_{x_\Delta}}\dot{\alpha}_{x_\Delta} + X_{\alpha_{x_\Delta}}\alpha_{x_\Delta} + X_{\dot{\beta}_{y_\Delta}}\dot{\beta}_{y_\Delta} \\ & + X_{\beta_{y_\Delta}}\beta_{y_\Delta} + X_{\dot{\beta}_{x_\Delta}}\dot{\beta}_{x_\Delta} + X_{\beta_{x_\Delta}}\beta_{x_\Delta} + X_{\dot{\beta}_{c_\Delta}}\dot{\beta}_{c_\Delta} \\ & + X_{\beta_{c_\Delta}}\beta_{c_\Delta} + X_{\Omega_\Delta}\Omega_\Delta + X_{\dot{z}}\dot{z} = X_{\theta_{y_\Delta}}\theta_{y_\Delta} \\ & + X_{\theta_{x_\Delta}}\theta_{x_\Delta} + X_{\theta_{c_\Delta}}\theta_{c_\Delta} \end{aligned} \tag{48}$$

$$Y_{\ddot y}\ddot y + Y_{\dot y}\dot y + Y_{\dot x}\dot x + Y_{\ddot\alpha_{x_\Delta}}\ddot\alpha_{x_\Delta} + Y_{\dot\alpha_{x_\Delta}}\dot\alpha_{x_\Delta}$$
$$+ Y_{\alpha_{x_\Delta}}\alpha_{x_\Delta} + Y_{\dot\alpha_{y_\Delta}}\dot\alpha_{y_\Delta} + Y_{\alpha_{y_\Delta}}\alpha_{y_\Delta} + Y_{\dot\beta_{y_\Delta}}\dot\beta_{y_\Delta}$$
$$+ Y_{\beta_{y_\Delta}}\beta_{y_\Delta} + Y_{\dot\beta_{x_\Delta}}\dot\beta_{x_\Delta} + Y_{\beta_{x_\Delta}}\beta_{x_\Delta} + Y_{\dot\beta_{c_\Delta}}\dot\beta_{c_\Delta} + Y_{\beta_{c_\Delta}}\beta_{c_\Delta}$$
$$+ Y_{\Omega_\Delta}\Omega_\Delta + Y_{\dot z}\dot z + Y_{\dot\alpha_{z_\Delta}}\dot\alpha_{z_\Delta} + Y_{\alpha_{z_\Delta}}\alpha_{z_\Delta} =$$
$$= Y_{\theta_{y_\Delta}}\theta_{y_\Delta} + Y_{\theta_{x_\Delta}}\theta_{x_\Delta} + Y_{\theta_{c_\Delta}}\theta_{c_\Delta} + Y_{\theta_{\Delta_t}}\theta_{\Delta_t} \tag{49}$$

$$Z_{\ddot z}\ddot z + Z_{\dot z}\dot z + Z_{\dot x}\dot x + Z_{\ddot\beta_{c_\Delta}}\ddot\beta_{c_\Delta}$$
$$+ Z_{\dot\beta_{c_\Delta}}\dot\beta_{c_\Delta} + Z_{\dot\alpha_{y_\Delta}}\dot\alpha_{y_\Delta} + Z_{\alpha_{y_\Delta}}\alpha_{y_\Delta} + Z_{\Omega_\Delta}\Omega_\Delta$$
$$+ Z_{\dot\beta_{x_\Delta}}\dot\beta_{x_\Delta} = Z_{\theta_{c_\Delta}}\theta_{c_\Delta} + Z_{\theta_{y_\Delta}}\theta_{y_\Delta} \tag{50}$$

The stability derivatives in the above three equations are defined by:

$$X_{\ddot x} = -\left(\frac{W}{g} + b\frac{W_b}{g}\right)$$
$$X_{\dot x} = -\Big[C_{D_f}\rho\pi R^2 V_0 + \beta_{c_s}^2\Omega_0\frac{A_2}{2} + \beta_{y_s}^2\Omega_0\frac{A_2}{2} + \beta_{y_s}\Omega_0\frac{A_2}{2}(\theta_{y_s}$$
$$+ \alpha_{y_0}) + v\left(\theta_{c_{s_R}}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) + \Omega_0 A_3\Big]$$
$$X_{\dot y} = -\beta_{c_s}\beta_{y_s}V_0E_2 - (\theta_{y_s} + \alpha_{y_0})\left(\beta_{c_s}V_0\frac{E_2}{2} + \beta_{x_s}\Omega_0\frac{A_2}{2}\right)$$
$$+ \beta_{x_s}\beta_{y_s}\Omega_0\frac{A_2}{2} - \beta_{c_s}v\frac{3}{2}E_2 + \beta_{c_s}\Omega_0\Big(\theta_{c_{s_R}}\frac{3A_2}{2}$$

$$X_{\ddot{\alpha}_{y_\Delta}} = - b \frac{W_b}{g} h$$

$$X_{\dot{\alpha}_{y_\Delta}} = h \left[\dot{X}_x + C_{D_f} \rho \pi R^2 V_0 \right]$$

$$X_{\alpha_{y_\Delta}} = -\Omega_0 \frac{A_z}{2} \left(V_0 \beta_{y_s} - v \right)$$

$$X_{\ddot{\alpha}_{x_\Delta}} = h \, X_{\dot{Y}}$$

$$X_{\alpha_{x_\Delta}} = - \beta_{c_s} \Omega_0^2 \frac{C_z}{2}$$

$$X_{\dot{\beta}_{y_\Delta}} = - \beta_{c_s} \Omega_0 \frac{C_z}{2} + \beta_{x_s} V_0 \frac{A_z}{8}$$

$$X_{\beta_{y_\Delta}} = - \beta_{y_s} V_0 \Omega_0 A_z + \Omega_0^2 \left(\Theta_{c_{s_R}} C_z - \frac{\Theta_d}{R} B_2 \right)$$
$$- V_0 \Omega_0 \frac{A_z}{2} \left(\Theta_{y_s} + \alpha_{y_0} \right) - \Omega_0 v \frac{3 A_z}{2}$$

$$X_{\dot{\beta}_{x_\Delta}} = \beta_{y_s} V_0 \frac{A_z}{8} - \Omega_0 \left(\Theta_{c_{s_R}} \frac{C_z}{2} - \frac{\Theta_d}{R} \frac{B_2}{2} \right) + V_0 \frac{3 A_z}{8} \left(\Theta_{y_s} \right.$$
$$\left. + \alpha_{y_0} \right) + v A_z$$

$$X_{\beta_{x_\Delta}} = - \beta_{c_s} \Omega_0^2 \frac{C_z}{2}$$

$$X_{\dot{\beta}_{c_\Delta}} = - \beta_{y_s} \Omega_0 \frac{3 C_z}{2} - V_0 \left(\Theta_{c_{s_R}} \frac{A_z}{2} - \frac{\Theta_d}{R} \frac{C_z}{2} \right)$$
$$+ \Omega_0 \frac{C_z}{2} \left(\Theta_{y_s} + \alpha_{y_0} \right)$$

$$X_{\beta_{c_\Delta}} = - \beta_{c_s} V_0 \Omega_0 A_z - \beta_{x_s} \Omega_0^2 \frac{C_z}{2}$$

$$X_{\Omega_\Delta} = -\beta_{c_s}\beta_{x_s}\Omega_o C_2 + \beta_{y_s}\Omega_o\left(\theta_{c_{s_R}} 2C_2 - \frac{\theta_d}{R} 2B_2\right) - (\theta_{y_s} + \alpha_{y_o})\beta_{y_s} V_o \frac{A_2}{2} - \beta_{c_s}^2 V_o \frac{A_2}{2} - \beta_{y_s}^2 V_o \frac{A_2}{2} - \beta_{y_s} v \frac{3}{2} A_2 - V_o A_3$$

$$X_{\dot{z}} = V_o\left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) - \Omega_o \frac{A_2}{2}(\theta_{y_s} + \alpha_{y_o}) + \beta_{y_s}\Omega_o \frac{3}{2} A_2$$

$$X_{\theta_{y_\Delta}} = X_{\alpha_{y_\Delta}}$$

$$X_{\theta_{x_\Delta}} = X_{\alpha_{x_\Delta}}$$

$$X_{\theta_{c_\Delta}} = -\left(\beta_{y_s}\Omega_o^2 C_2 - V_o v \frac{E_2}{2}\right)$$

$$Y_{\ddot{y}} = -\frac{W}{g} - b\frac{W_b}{g}$$

$$Y_{\dot{y}} = -\beta_{c_s}\beta_{x_s} V_o E_2 - \beta_{c_s}^2 \Omega_o \frac{A_2}{2} - \beta_{x_s}^2 \Omega_o \frac{A_2}{2} - \beta_{y_s} V_o\left(\theta_{c_{s_R}}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) - v\left(\theta_{c_{s_R}}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) - \Omega_o A_3 - \Omega_{o_x} A_4$$

$$Y_{\dot{x}} = (\theta_{y_s} + \alpha_{y_o})(\beta_{c_s} V_o E_2 + \beta_{x_s}\Omega_o A_2) - (3\beta_{c_s}\Omega_o)\left(\theta_{c_{s_R}}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right) - \beta_{x_s} V_o\left(\theta_{c_{s_R}} E_2 - \frac{\theta_d}{R} A_2\right) + \beta_{c_s}\beta_{y_s} V_o 2E_2 + \beta_{x_s}\beta_{y_s}\Omega_o\frac{A_2}{2} + \beta_{c_s} v \frac{3}{2} E_2 + \theta_{o_x} V_o E_4$$

$$Y_{\ddot{\alpha}_{x_\Delta}} = -b\frac{W_b}{g}h$$

$$Y_{\dot{\alpha}_{x_\Delta}} = h\left(Y_{\dot{y}} + \Omega_{o_x} A_4\right)$$

$$Y_{\alpha_{x_\Delta}} = \beta_{y_S} V_o \Omega_o \frac{A_2}{2} + \Omega_o v \frac{A_2}{2}$$

$$Y_{\ddot{\alpha}_{y_\Delta}} = h\left(Y_{\dot{x}} - \Theta_{o_x} V_o E_4\right)$$

$$Y_{\alpha_{y_\Delta}} = \beta_{c_S}\left(V_o^2\frac{E_2}{2} + \Omega_o^2\frac{C_2}{2}\right) + \beta_{x_S} V_o \Omega_o A_2$$

$$Y_{\dot{\beta}_{y_\Delta}} = \beta_{y_S} V_o \frac{7A_2}{8} - \Omega_o\left(\Theta_{c_{S_R}}\frac{C_2}{2} - \frac{\Theta_d}{R}\frac{B_2}{2}\right) + vA_2 + V_o\frac{A_2}{8}\left(\Theta_{y_S} + \alpha_{y_o}\right)$$

$$Y_{\beta_{y_\Delta}} = \beta_{c_S} V_o^2 E_2 - \beta_{c_S}\Omega_o^2\frac{C_2}{2} + \beta_{x_S} V_o\Omega_o\frac{A_2}{2}$$

$$Y_{\dot{\beta}_{x_\Delta}} = \beta_{c_S}\Omega_o\frac{C_2}{2} + \beta_{x_S} V_o\frac{5A_2}{8}$$

$$Y_{\beta_{x_\Delta}} = -V_o^2\left(\Theta_{c_{S_R}}\frac{E_2}{2} - \frac{\Theta_d}{R}\frac{A_2}{2}\right) - \Omega_o^2\left(\Theta_{c_{S_R}} C_2 - \frac{\Theta_d}{R}B_2\right) + V_o\Omega_o A_2\left(\Theta_{y_S} + \alpha_{y_o}\right) + \Omega_o\frac{A_2}{2}\left(\beta_{y_S} V_o + v3\right)$$

$$Y_{\dot{\beta}_{c_\Delta}} = \beta_{c_S} V_o\frac{3A_2}{2} + \beta_{x_S}\Omega_o\frac{3C_2}{2}$$

$$Y_{\beta_{c_\Delta}} = \left(\Theta_{y_S} + \alpha_{y_o}\right)\left(V_o^2\frac{E_2}{2} + \Omega_o^2\frac{C_2}{2}\right) - 3V_o\Omega_o\left(\Theta_{c_{S_R}}\frac{A_2}{2} - \frac{\Theta_d}{R}\frac{C_2}{2}\right) + V_o E_2\left(\beta_{y_S} V_o + v\frac{3}{2}\right) - \beta_{y_S}\Omega_o^2\frac{C_2}{2}$$

$$Y_{\Omega_\Delta} = (\theta_{y_s} + \alpha_{y_0})(\beta_{c_s}\Omega_0 C_2 + \beta_{x_s} V_0 A_2) - 3\beta_{c_s} V_0\left(\theta_{c_{s_R}}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right) - 2\beta_{x_s}\Omega_0\left(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R}B_2\right) - \beta_{c_s}\beta_{y_s}\Omega_0 C_2 + \beta_{x_s}\left(\frac{3}{2}A_2 v + \beta_{y_s} V_0 \frac{A_2}{2}\right) + \theta_{0_t} 2\Omega_{0_t} C_4 - v_t A_4$$

$$Y_{\dot{y}} = -\beta_{c_s} V_0 \frac{3E_2}{2} - \beta_{x_s}\Omega_0 \frac{3A_2}{2}$$

$$Y_{\dot{\alpha}_{z_\Delta}} = \Omega_{0_t} l_t A_4$$

$$Y_{\alpha_{z_\Delta}} = \Omega_{0_t} V_0 A_4$$

$$Y_{\theta_{y_\Delta}} = Y_{\alpha_{y_\Delta}}$$

$$Y_{\theta_{x_\Delta}} = Y_{\alpha_{x_\Delta}}$$

$$Y_{\theta_{c_\Delta}} = \beta_{x_s}\left(V_0^2 \frac{E_2}{2} + \Omega_0^2 C_2\right) + 3\beta_{c_s} V_0 \Omega_0 A_2/2$$

$$Y_{\theta_{\Delta_t}} = -\Omega_{0_t}^2 C_4 - V_0^2 \frac{E_4}{2}$$

$$Z_{\ddot{y}} = -\frac{W}{g} - b\frac{W_b}{g}$$

$$Z_{\dot{y}} = -\Omega_0 A_2$$

$$Z_{\dot{x}} = -V_0\left(\theta_{c_{s_R}} E_2 - \theta_d A_2/R\right) + (\theta_{y_s} + \alpha_{y_0})\Omega_0 A_2$$

$$Z_{\ddot{\beta}_{c_\Delta}} = b J_b$$

$$Z_{\dot{\beta}_{c_\Delta}} = \Omega_o C_2$$

$$Z_{\dot{\alpha}_{y_\Delta}} = h Z_{\dot{x}}$$

$$Z_{\alpha_{y_\Delta}} = \Omega_o V_o A_2$$

$$Z_{\Omega_\Delta} = -2\Omega_o \left(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2\right) + \left(\theta_{y_s} + \alpha_{y_o}\right) V_o A_2 + v A_2$$

$$Z_{\dot{\beta}_{x_\Delta}} = V_o \frac{A_2}{2}$$

$$Z_{\theta_{c_\Delta}} = V_o^2 \frac{E_2}{2} + \Omega_o^2 C_2$$

$$Z_{\theta_{y_\Delta}} = \Omega_o V_o A_2$$

SECTION IV

EQUATIONS OF PITCHING, ROLLING, AND YAWING MOTION

4.1 THE BLADE MOMENTS

Inasmuch as the inclination of the flapping hinge is being ignored in the present development, the only moments which the rotor can transmit to the helicopter are associated with the eccentricity of the flapping hinges.

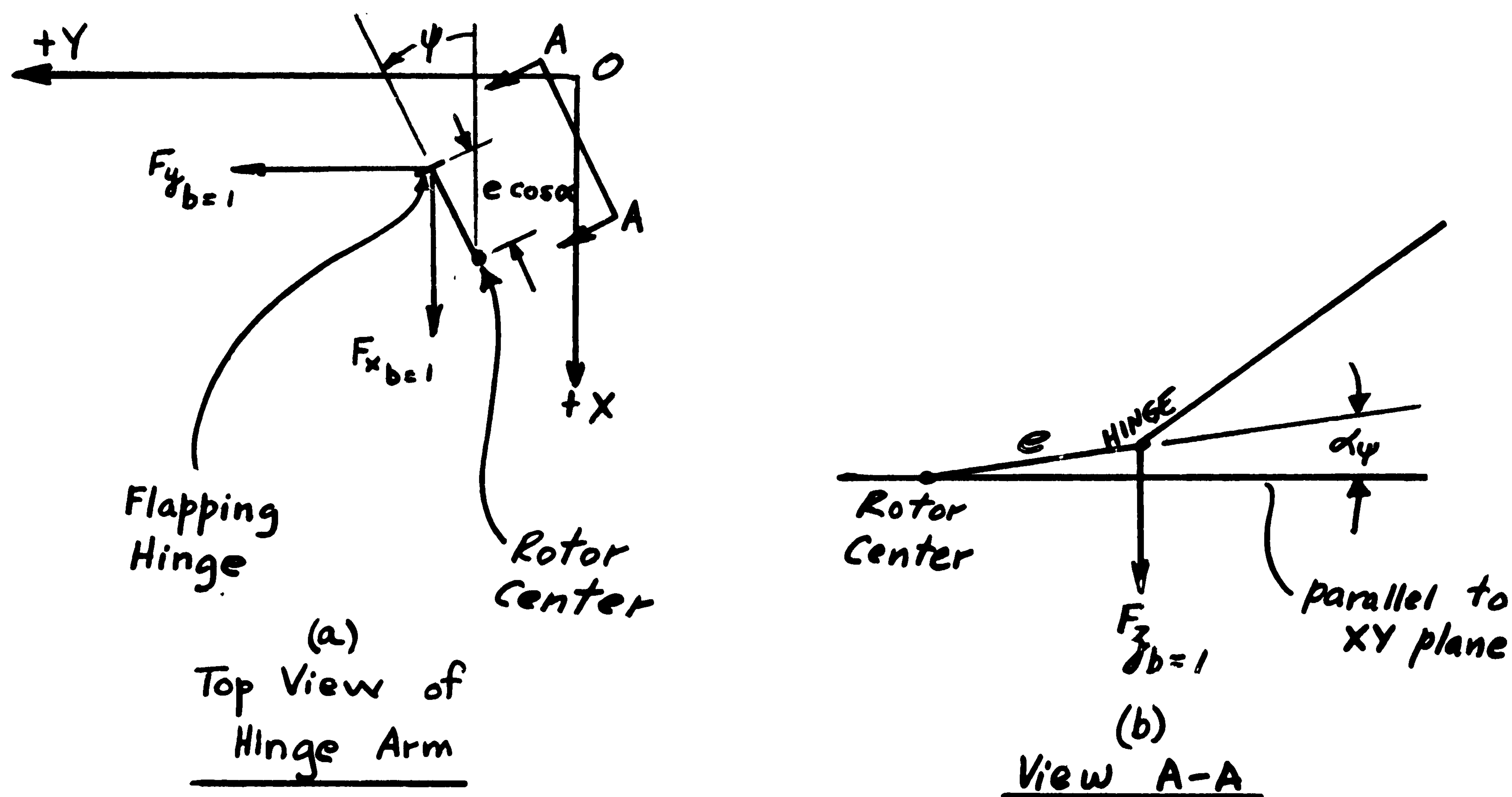

Figure 7

Forces at the Flapping Hinge

The views above are similar to those shown in Figure 4 previously, but here attention is confined to the region of the rotor center, hinge arm, and flapping hinge. Shown acting on the rigid hinge arm, at the hinge point, are the three blade force components F_x , F_y , and F_z . In transferring these force components to the rotor center, there must be introduced the moments associated with the distance through which the forces are moved. The moments are grouped into two components, M_y and M_x , given by (see similar development in equations 107, 109, 112, 113 and 114 in Ref. 1):

$$M_{y_{b=1}} = e\alpha_\psi F_{x_{b=1}} - eF_{z_{b=1}} \cos\psi \qquad (51)$$

$$M_{x_{b=1}} = e\alpha_\psi F_{y_{b=1}} + eF_{z_{b=1}} \sin\psi \qquad (52)$$

The algebraic signs in these equations obey the conventions established here in Figure 1. The angle α_ψ, shown in Figure 4(a) and 7(b), can be expressed in terms of its "components" as

$$\alpha_\psi = \alpha_y \cos\psi - \alpha_x \sin\psi \qquad (53)$$

The minus sign in equation (53) is a consequence of the position of the flapping hinge in the first quadrant of blade azimuth (see Figure 7b); as shown, the "roll" component of α_ψ corresponds to a left roll, which is taken here to be negative. Expanded, equation (53) gives

$$\alpha_\psi = \alpha_{y_0} \cos\psi_0 - \left[(\alpha_{y_0}\psi_\Delta + \alpha_{x_\Delta}) \cdot \sin\psi_0 - \alpha_{y_\Delta} \cos\psi_0\right] \qquad (54)$$

The blade moment components M_y and M_x (equations 51-52) can be evaluated by means of equations (34), (35), (36), and (54).

4.2 THE ROTOR MOMENT

Instead of carrying out the evaluation of equations (51) and (52) for each blade and then summing the contribution of the b blades in the rotor, considerable reduction in effort is achieved by (a) taking each part of the blade element force expression, (b) performing the operation called for by equations (51) and (52) on (a) above, (c) summing the contribution of b blade elements of the result in (b) above, and then integrating over the blade span. This was the procedure employed in the hovering analysis also (see Ref. 1 - Art. 4.2). As explained in Appendix II and III of Reference (1), the process of summing the contribution of b blades generally corresponds to averaging each part of the blade element moment contribution over one complete azimuthal revolution and multiplying by the number of blades.

An example of the above procedure is shown in the following. From equation (34), and the first part of equation (51), there is the term $e\alpha \int (-\ddot{x}_b \, dm_b) =$

$$e\left[\alpha_{y_0}\cos\psi_0 - \left[(\alpha_{y_0}\psi_\Delta + \alpha_{x_\Delta})\sin\psi_0 - \alpha_{y_\Delta}\cos\psi_0\right]\right]\left\{-\int_e^R\left[\left(\ddot{x} + h\ddot{\alpha}_{y_\Delta} + r\left[(-\dot{\Omega}_\Delta - \psi_\Delta\Omega_0^2)\sin\psi_0 + (\Omega_0^2 + 2\Omega_\Delta\Omega_0)\cos\psi_0\right]\right]dm_b\right\}$$

Summing first for b blades (note that

$$\frac{b}{2\pi}\int_0^{2\pi}\cos\psi_0 \, d\psi_0 = \frac{b}{2\pi}\int_0^{2\pi}\sin\psi_0 \, d\psi_0 = \frac{b}{2\pi}\int_0^{2\pi}\sin\psi_0\cos\psi_0 \, d\psi_0 = 0)$$

this yields (see also Appendix II - Ref. 1),

$$-eb\int_e^R\left[\alpha_{y_0}(\Omega_0^2 + 2\Omega_\Delta\Omega_0)\frac{r}{2} + \alpha_{y_\Delta}\Omega_0^2\frac{r}{2}\right]\bar{\rho}\, dr$$

Integration over the blade span yields

$$-\frac{e}{2}\left[\alpha_{y_0}(\Omega_0^2 + 2\Omega_\Delta\Omega_0) + \alpha_{y_\Delta}\Omega_0^2\right]L_4 \tag{55}$$

Proceeding in like manner, the total rotor moments were obtained as, omitting steady-state terms,

$$\begin{aligned} M_{y_R} = e\Big\{&(\dot{x} + h\dot{\alpha}_{y_\Delta})\Big[\alpha_{y_0}\beta_{c_s}\Omega_0\frac{A_2}{8}(\theta_{y_s} + \alpha_{y_0}) + \beta_{c_s}\Omega_0 A_2 \cdot \\ &\cdot\left(\frac{1}{2} + \alpha_{y_0}\beta_{y_s}\frac{7}{8}\right) - \alpha_{y_0}\beta_{x_s}v\frac{\epsilon_2}{8} + \beta_{x_s}V_0\frac{\epsilon_2}{4}\Big] + (\dot{y} + h\dot{\alpha}_{x_\Delta}) \cdot \\ &\cdot\Big[\alpha_{y_0}\beta_{c_s}\left(\beta_{c_s}V_0\frac{\epsilon_2}{2} + \beta_{x_s}\Omega_0\frac{3A_2}{8}\right) + \left(\alpha_{y_0}\beta_{y_s}V_0\frac{\epsilon_2}{8} - \alpha_{y_0}v\frac{\epsilon_2}{8}\right) \cdot \\ &\cdot(\theta_{y_s} + \alpha_{y_0}) + V_0\frac{\epsilon_2}{4}(\theta_{y_s} + \alpha_{y_0}) - \Omega_0(\alpha_{y_0}\beta_{y_s} + 1)\Big(\theta_{c_{s_R}}A_2 \\ &- \frac{\theta_d}{R}C_2\Big) + \alpha_{y_0}\beta_{y_s}\frac{\epsilon_2}{8}(\beta_{y_s}V_0 3 + 5v) + \alpha_{y_0}V_0\Big(\beta_{x_s}^2\frac{\epsilon_2}{8} \\ &+ \frac{\epsilon_3}{4}\Big) + \frac{\epsilon_2}{2}\left(\beta_{y_s}\frac{V_0}{2} + v\right)\Big] - \theta_{y_\Delta}\left[\alpha_{y_0}\beta_{c_s}\Omega_0 V_0\frac{A_2}{8}\right] + \end{aligned}$$

$$
\begin{aligned}
&+ \alpha_{y_\Delta}\Big[-\beta_{c_s}\Omega_o V_o \tfrac{A_2}{8}(\theta_{y_s} - 2\alpha_{y_o}) - \Omega_o^2 \tfrac{L_4}{2} + \beta_{c_s}\Omega_o^2\big(\theta_{c_{s_R}}\tfrac{C_2}{2} - \tfrac{\theta_d}{R}\tfrac{B_2}{2}\big) \\
&- \beta_{c_s}\Omega_o A_2\big(\beta_{y_s}V_o \tfrac{7}{8} + \upsilon\tfrac{1}{2}\big) - \beta_{x_s}\big(\beta_{y_s}\Omega_o^2\tfrac{C_2}{2} - \upsilon V_o\tfrac{E_2}{8}\big)\Big] + \dot\beta_{x_\Delta}\Big[2\Omega_o\tfrac{L_4}{2} \\
&- \alpha_{y_o}\tfrac{1}{8}(\beta_{c_s}V_o A_2 + \beta_{x_s}\Omega_o C_2)\Big] + \beta_{x_\Delta}\Big[\alpha_{y_o}\beta_{y_s}\Omega_o^2\tfrac{C_2}{2} - V_o\tfrac{E_2}{8}(\alpha_{y_o}\upsilon V_o \\
&+ V_o) + \Omega_o^2\tfrac{C_2}{2}\Big] + \ddot\beta_{y_\Delta}\Big[\tfrac{L_4}{2}\Big] + \dot\beta_{y_\Delta}\Big[\alpha_{y_o}\beta_{y_s}\Omega_o\tfrac{5C_2}{8} \\
&+ \alpha_{y_o}V_o\big(\theta_{c_{s_R}}\tfrac{A_2}{8} - \tfrac{\theta_d}{R}\tfrac{C_2}{8}\big) + \Omega_o\tfrac{C_2}{2} - \alpha_{y_o}\Omega_o\tfrac{C_2}{8}(\theta_{y_s} + \alpha_{y_o})\Big] \\
&+ \beta_{y_\Delta}\Big[\alpha_{y_o}\Omega_o\big(\beta_{c_s}V_o\tfrac{7}{8}A_2 + \beta_{x_s}\Omega_o\tfrac{C_2}{2}\big) - \Omega_o^2\tfrac{L_4}{2}\Big] + \dot\beta_{c_\Delta}\Big[\alpha_{y_o}\big(\beta_{c_s}\Omega_o\tfrac{C_2}{2} - \beta_{x_s}V_o\tfrac{A_2}{8}\big)\Big] \\
&+ \beta_{c_\Delta}\Big[-\alpha_{y_o}\Omega_o^2\big(\theta_{c_{s_R}}\tfrac{C_2}{2} - \tfrac{\theta_d}{R}\tfrac{B_2}{2}\big) + \alpha_{y_o}\Omega_o V_o\tfrac{A_2}{8}(\theta_{y_s} + \alpha_{y_o} + 7\beta_{y_s}) \\
&+ \Omega_o\tfrac{A_2}{2}(\alpha_{y_o}\upsilon + V_o)\Big] + \theta_{c_\Delta}\Big[-\alpha_{y_o}\beta_{c_s}\Omega_o^2\tfrac{C_2}{2}\Big] + \dot z\Big[-\alpha_{y_o}\beta_{c_s}\Omega_o\tfrac{A_2}{2} \\
&+ \alpha_{y_o}\beta_{x_s}V_o\tfrac{E_2}{8}\Big] + \dot\Omega_\Delta\Big[\beta_{x_s}\tfrac{L_4}{2}\Big] + \Omega_\Delta\Big[\alpha_{y_o}\Omega_o L_4 - \alpha_{y_o}\beta_{c_s}\Omega_o\big(\theta_{c_{s_R}}C_2 \\
&- \tfrac{\theta_d}{R}B_2\big) + \alpha_{y_o}\beta_{c_s}V_o\tfrac{A_2}{8}(\theta_{y_s} + \alpha_{y_o}) + \alpha_{y_o}\beta_{c_s}\tfrac{A_2}{2}\big(\beta_{y_s}V_o\tfrac{7}{4} + \upsilon\big) - \beta_{y_s}\Omega_o\big(L_4 \\
&- \alpha_{y_o}\beta_{x_s}C_2\big) + \beta_{x_s}\Omega_o C_2 + \beta_{c_s}V_o\tfrac{A_2}{2}\Big] + \theta_{x_\Delta}\Big[-\alpha_{y_o}\big(\beta_{y_s}\Omega_o^2\tfrac{C_2}{2} - V_o\upsilon\tfrac{E_2}{8}\big) \\
&+ V_o^2\tfrac{E_2}{8} - \Omega_o^2\tfrac{C_2}{2}\Big] + \alpha_{x_\Delta}\Big[\alpha_{y_o}\big(\beta_{y_s}\Omega_o^2\tfrac{C_2}{2} - V_o\upsilon\tfrac{E_2}{8}\big) + V_o^2\tfrac{E_2}{8} \\
&+ \Omega_o^2\tfrac{C_2}{2} - \beta_{c_s}\beta_{x_s}V_o\Omega_o\tfrac{A_2}{8} + (\beta_{y_s}\Omega_o V_o - \Omega_o\upsilon)\big(\theta_{c_{s_R}}\tfrac{A_2}{2} \\
&- \tfrac{\theta_d}{R}\tfrac{C_2}{2}\big) - \big(\beta_{y_s}\Omega_o^2\tfrac{C_2}{2} - V_o\upsilon\tfrac{3E_2}{8}\big)(\theta_{y_s} + \alpha_{y_o}) + \beta_{y_s}V_o\upsilon\tfrac{E_2}{8} \\
&+ \beta_{y_s}^2\Omega_o^2\tfrac{C_2}{2} + \upsilon^2\tfrac{E_2}{2} - V_o^2\tfrac{3E_3}{8} - \Omega_o^2\tfrac{C_2}{2}\Big]\Big\} \qquad (56)
\end{aligned}
$$

$$
\begin{aligned}
M_{x_R} = e\Big\{&(\dot z + h\dot\alpha_{y_\Delta})\Big[\alpha_{y_o}\beta_{c_s}\big(\beta_{c_s}V_o E_2 + \beta_{x_s}\Omega_o\tfrac{9A_2}{8} + \alpha_{y_o}\beta_{x_s}^2 V_o\tfrac{E_2}{4} \\
&- \alpha_{y_o}\beta_{y_s}\Omega_o\big(\theta_{c_{s_R}}\tfrac{A_2}{2} - \tfrac{\theta_d}{R}\tfrac{C_2}{2}\big) + (\theta_{y_s} + \alpha_{y_o})\big(\alpha_{y_o}\beta_{y_s}V_o\tfrac{E_2}{4} + \alpha_{y_o}\upsilon\tfrac{E_2}{8} \\
&+ V_o\tfrac{3E_2}{4}\big) + \alpha_{y_o}\beta_{y_s}\tfrac{E_2}{4}\big(\beta_{y_s}V_o 3 + \upsilon\tfrac{7}{2}\big) - \alpha_{y_o}V_o\tfrac{E_3}{4} - \Omega_o\big(\theta_{c_{s_R}}A_2 \\
&- \tfrac{\theta_d}{R}C_2\big) + \tfrac{E_2}{2}\big(\beta_{y_s}\tfrac{V_o}{2} + \upsilon\big)\Big] + (\dot y + h\dot\alpha_{x_\Delta})\Big[-\alpha_{y_o}\beta_{c_s}V_o\big(\theta_{c_{s_R}}\tfrac{E_2}{2} \\
&- \tfrac{\theta_d}{R}\tfrac{A_2}{2}\big) + (\theta_{y_s} + \alpha_{y_o})\alpha_{y_o}\big(\beta_{c_s}\Omega_o\tfrac{A_2}{8} + \beta_{x_s}V_o\tfrac{E_2}{8}\big) - \alpha_{y_o}\beta_{y_s}\cdot \\
&\cdot\big(\beta_{c_s}\Omega_o\tfrac{A_2}{8} + \beta_{x_s}V_o\tfrac{E_2}{4}\big) - \alpha_{y_o}\beta_{x_s}\Omega_o\big(\theta_{c_{s_R}}\tfrac{A_2}{2} - \tfrac{\theta_d}{R}\tfrac{C_2}{2}\big)
\end{aligned}
$$

$$- \alpha_{y_0} \beta_{x_s} v \frac{E_2}{8} - \beta_{x_s} V_0 \frac{E_2}{4} - \beta_{c_s} \Omega_0 \frac{A_2}{2}] - \theta_{y_\Delta} [\alpha_{y_0} V_0 \frac{E_2}{8} (\beta_{y_s} V_0$$

$$+ v) + V_0^2 \frac{3E_2}{8} + \Omega_0^2 \frac{C_2}{2}] - \alpha_{y_\Delta} [- \alpha_{y_0} V_0 \frac{E_2}{8} (\beta_{y_s} V_0 + v) - V_0^2 \frac{3E_2}{8} - \Omega_0^2 \frac{C_2}{2}$$

$$- \beta_{c_s} V_0 (\beta_{c_s} V_0 \frac{E_2}{2} + \beta_{x_s} \Omega_0 \frac{9A_2}{8}) - \beta_{x_s}^2 (V_0^2 \frac{E_2}{8} + \Omega_0^2 \frac{C_2}{2}) + (\theta_{cs_R} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2}) \cdot$$

$$\cdot (\beta_{y_s} V_0 \Omega_0 + v \Omega_0) - (\theta_{y_s} + \alpha_{y_0}) (\beta_{y_s} V_0 + v) V_0 \frac{E_2}{8} - \beta_{y_s} V_0 \frac{E_2}{8} (7 v + \beta_{y_s} V_0 3)$$

$$- v^2 \frac{E_2}{2} + V_0^2 \frac{E_3}{8} + \Omega_0^2 \frac{C_3}{2}] + \ddot{\beta}_{x_\Delta} [\frac{L_4}{2}] + \dot{\beta}_{x_\Delta} [\Omega_0 \frac{C_2}{2} (1 - \alpha_{y_0} \beta_{y_s} / 4)$$

$$- \alpha_{y_0} V_0 (\theta_{cs_R} \frac{A_2}{8} - \frac{\theta_d}{R} \frac{C_2}{8}) + \alpha_{y_0} \Omega_0 \frac{C_2}{8} (\theta_{y_s} + \alpha_{y_0})] + \beta_{x_\Delta} [- \Omega_0^2 \frac{L_4}{2}$$

$$+ \alpha_{y_0} \beta_{c_s} V_0 \Omega_0 \frac{9}{8} A_2 + \alpha_{y_0} \beta_{x_s} (V_0^2 \frac{E_2}{4} + \Omega_0^2 C_2)] + \dot{\beta}_{y_\Delta} [\alpha_{y_0} \beta_{c_s} V_0 \frac{7A_2}{8}$$

$$+ \alpha_{y_0} \beta_{x_s} \Omega_0 \frac{7C_2}{8} - \Omega_0 L_4] + \beta_{y_\Delta} [\alpha_{y_0} V_0 \frac{E_2}{4} (\beta_{y_s} V_0 3 + v \frac{7}{2}) - \alpha_{y_0} V_0 \Omega_0 \cdot$$

$$\cdot (\theta_{cs_R} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2}) + \alpha_{y_0} \Omega_0^2 \frac{C_2}{4} (\theta_{y_s} + \alpha_{y_0}) + V_0^2 \frac{E_2}{8} - \Omega_0^2 \frac{C_2}{2}] + \dot{\beta}_{c_\Delta} [\alpha_{y_0} v A_2$$

$$+ V_0 \frac{A_2}{2} + \alpha_{y_0} \beta_{y_s} V_0 \frac{7A_2}{8} - \alpha_{y_0} \Omega_0 (\theta_{cs_R} \frac{C_2}{2} - \frac{\theta_d}{R} \frac{B_2}{2}) - \alpha_{y_0} V_0 \frac{A_2}{8} (\theta_{y_s} + \alpha_{y_0})]$$

$$+ \beta_{c_\Delta} [\alpha_{y_0} V_0 (\beta_{c_s} V_0 E_2 + \Omega_0 \beta_{x_s} \frac{9}{8} A_2)] + \theta_{c_\Delta} [- \alpha_{y_0} \Omega_0 \frac{A_2}{2} (\beta_{y_s} V_0 + v) - \Omega_0 V_0 A_2]$$

$$+ \dot{z} [- \alpha_{y_0} V_0 \frac{E_2}{8} (7 \beta_{y_s} + \theta_{y_s} + \alpha_{y_0}) + \alpha_{y_0} \Omega_0 (\theta_{cs_R} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2}) - V_0 \frac{E_2}{2}$$

$$- \alpha_{y_0} v E_2] - \dot{\Omega}_\Delta [\frac{L_4}{2} (- \alpha_{y_0} - \beta_{y_s})] + \Omega_\Delta [\alpha_{y_0} \beta_{c_s} \beta_{x_s} V_0 \frac{9A_2}{8}$$

$$+ \alpha_{y_0} \beta_{x_s}^2 \Omega_0 C_2 - \alpha_{y_0} (\beta_{y_s} V_0 + v) (\theta_{cs_R} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2}) - \beta_{x_s} \Omega_0 L_4 - V_0 (\theta_{cs_R} A_2$$

$$- \frac{\theta_d}{R} C_2) + \Omega_0 C_2 (\theta_{y_s} + \alpha_{y_0}) - \beta_{y_s} \Omega_0 C_2 - \alpha_{y_0} \Omega_0 C_3] + \theta_{x_\Delta} [- \alpha_{y_0} \beta_{c_s} \Omega_0 V_0 \frac{5A_2}{8}$$

$$- \alpha_{y_0} \beta_{x_s} (V_0^2 \frac{E_2}{8} + \Omega_0^2 \frac{C_2}{2})] + \alpha_{x_\Delta} [\alpha_{y_0} \beta_{c_s} \Omega_0 V_0 \frac{5A_2}{8} + \alpha_{y_0} \beta_{x_s} (V_0^2 \frac{E_2}{8} + \Omega_0^2 \frac{C_2}{2})$$

$$- \Omega_0^2 \frac{L_4}{2} + \beta_{y_s} \beta_{c_s} V_0 \Omega_0 \frac{3A_2}{8} + \beta_{c_s} V_0^2 (\theta_{cs_R} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2}) + \beta_{c_s} \Omega_0^2 \cdot$$

$$\cdot (\theta_{cs_R} \frac{C_2}{2} - \frac{\theta_d}{R} \frac{B_2}{2}) - (\theta_{y_s} + \alpha_{y_0}) (\beta_{c_s} \Omega_0 V_0 \frac{7A_2}{8} + \beta_{x_s} V_0^2 \frac{3E_2}{8} + \beta_{x_s} \Omega_0^2 \frac{C_2}{2})$$

$$- \beta_{c_s} v \Omega_0 \frac{A_2}{2} + \beta_{x_s} \Omega_0 V_0 (\theta_{cs_R} A_2 - \frac{\theta_d}{R} C_2) - \beta_{x_s} v V_0 \frac{5E_2}{8} - \beta_{y_s} \beta_{x_s} V_0^2 \frac{E_2}{4}$$

$$+ \beta_{x_s} \beta_{y_s} \Omega_0^2 \frac{C_2}{2}] \}$$

(57)

4.3 THE FUSELAGE AND TAIL MOMENTS

The contribution of the helicopter fuselage and fixed tail surfaces to the dynamics of cruising flight is a matter of conjecture. In the present development, it was assumed that the longitudinal (pitching) motion of the aircraft may be affected by moments arising with the fuselage and tail surfaces, while lateral (rolling and yawing) motion was not. The latter may be assumed to be a consequence of tail effects cancelling fuselage effects in the lateral axes.

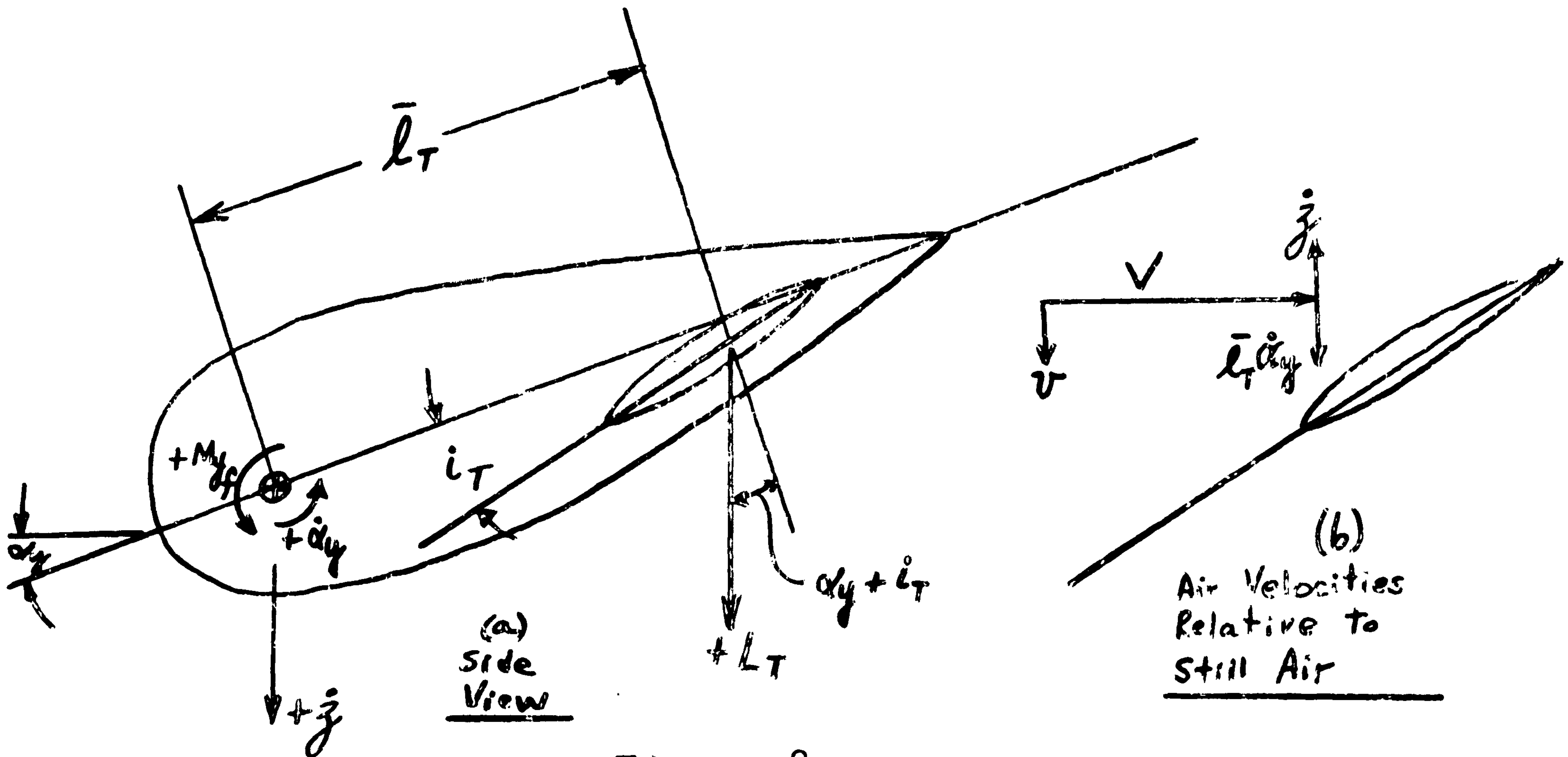

Figure 8

Longitudinal Forces and Moments of Fuselage and Tail Surfaces

The fuselage moment is usually written as

$$M_{y_f} = C_{m_f} \frac{\rho V_o^2}{2} \pi R^3 \tag{58}$$

During a transient,

$$M_{y_{f\Delta}} = \frac{\partial M_{y_f}}{\partial \alpha_y} \alpha_{y_\Delta}$$

$$= \frac{\partial C_{m_f}}{\partial \alpha_y} \frac{\rho V_o^2}{2} \pi R^3 \alpha_{y_\Delta} \tag{59}$$

The tail provides several moment terms, as follows:

$$L_T = C_{L_T} \rho \frac{V_0^2}{2} S_T \tag{60}$$

$$M_{y_T} = -L_T \bar{l}_T \cos(\alpha_y + i_T) = -L_T \bar{l}_T$$

$$= -\bar{a}_T \left(\alpha_y + i_T - \frac{v}{V_0}\right) \rho \frac{V_0^2}{2} \bar{S}_T \bar{l}_T \tag{61}$$

$$M_{y_{T_{\Delta 1}}} = \frac{\partial M_{y_{T_\alpha}}}{\partial \alpha_y} \alpha_{y_\Delta}$$

$$= -\bar{a}_T \rho \frac{V_0^2}{2} \bar{S}_T \bar{l}_T \alpha_{y_\Delta} \tag{62}$$

$$= -M_{y_{T_\alpha}} \alpha_{y_\Delta} \tag{63}$$

As shown in Figure 8(b), the tail lift also depends on $\dot{\alpha}_y$ and $\dot{z}$, since these motions affect the tail angle of attack. Thus, the motion $\bar{l}_T \dot{\alpha}_y$ increases the angle of attack, so

$$\Delta L_T = \bar{a}_T \Delta\alpha \rho \frac{V_0^2}{2} \bar{S}_T$$

$$= \bar{a}_T \frac{\bar{l}_T}{V_0} \dot{\alpha}_{y_\Delta} \rho \frac{V_0^2}{2} \bar{S}_T \tag{64}$$

Thus,

$$M_{y_{T_{\Delta 2}}} = \frac{-\bar{a}_T \bar{l}_T}{V_0} \dot{\alpha}_{y_\Delta} \rho \frac{V_0^2}{2} \bar{S}_T \bar{l}_T \tag{65}$$

$$= -M_{y_{T_\alpha}} \frac{\bar{l}_T}{V_0} \dot{\alpha}_{y_\Delta} \tag{66}$$

Also,

$$\Delta L_T = -\bar{a}_T \frac{\dot{z}}{V_0} \rho \frac{V_0^2}{2} \bar{S}_T \tag{67}$$

and $$M_{y_{T_{\Delta_{3}}}} = \bar{a}_T \frac{\dot{z}}{V_0} \frac{\rho V_0^2}{2} \bar{S}_T \bar{l}_T$$

$$= M_{y_{T_\alpha}} \dot{z}/V_0 \qquad (68)$$

Thus, the tail surfaces contribute to the longitudinal moments acting on the helicopter as follows (from 63, 66, and 68):

$$M_{y_{T_\Delta}} = - M_{y_{T_\alpha}} \left(\alpha_{y_\Delta} + \frac{\bar{l}_T}{V_0} \dot{\alpha}_{y_\Delta} - \frac{\dot{z}}{V_0} \right) \qquad (69)$$

4.4 THE EQUATION OF HELICOPTER PITCHING MOTION

The free-body helicopter with longitudinal forces and moments is illustrated in Figure 9 below.

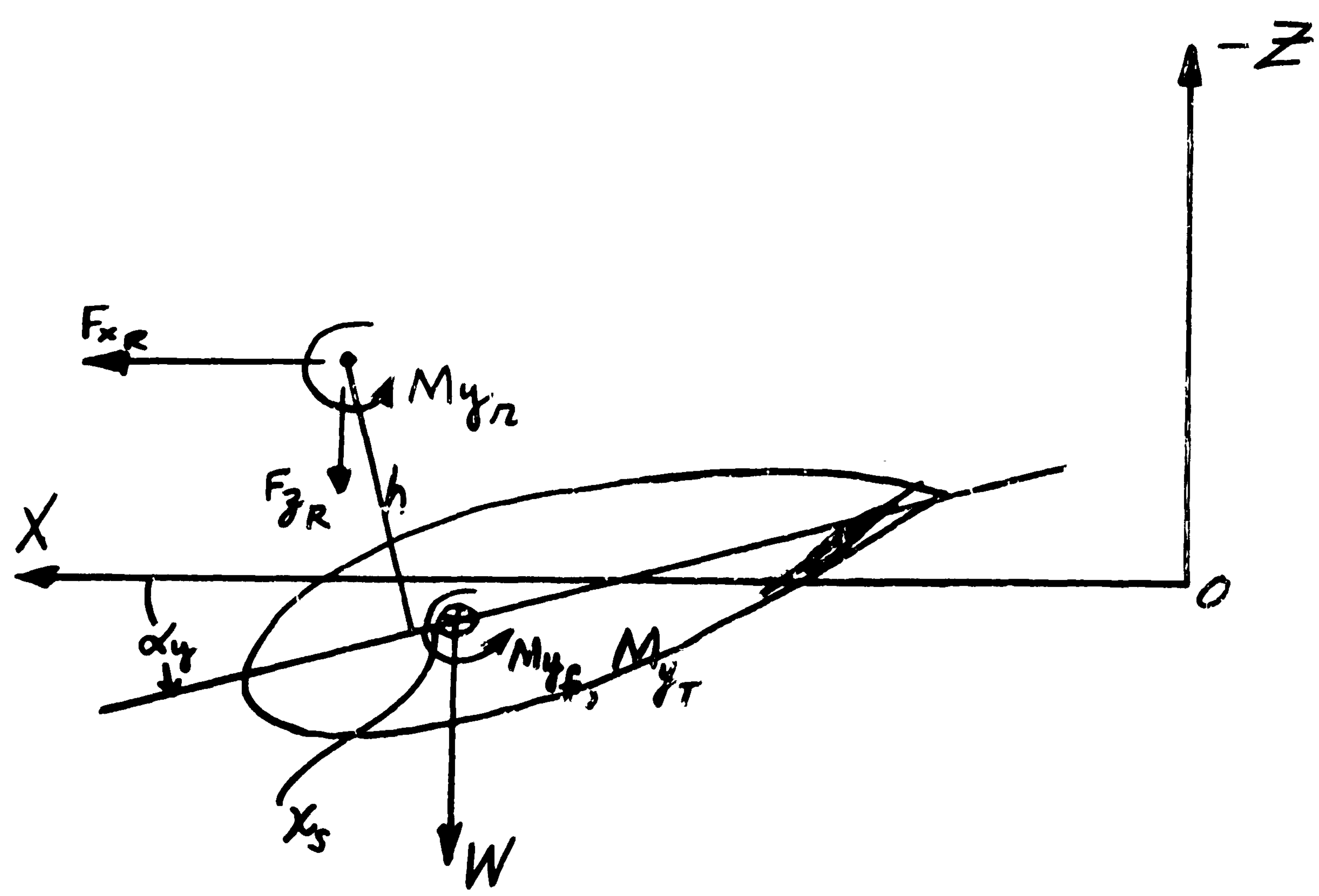

Figure 9

Longitudinal Forces and Moments

Thus,

$$F_{x_R} h - F_{x_R} x_s \alpha_y + F_{z_R} h \alpha_y + F_{z_R} x_s + M_{y_R} + M_{y_f} + M_{y_T} = I_y \ddot{\alpha}_{y_\Delta} \qquad (70)$$

Of interest here are the transient portions of F_{x_R}, F_{z_R}, M_{y_R}, M_{y_F}, and M_{y_T}, and the steady-state portions of F_{x_R} and F_{z_R} (see second and third terms in equation 70). These terms are given by equations (37), (39), (56), (59), and (69). On this basis, the pitching motion equation becomes

$$
\begin{aligned}
& P_{\ddot{\alpha}_{y_\Delta}} \ddot{\alpha}_{y_\Delta} + P_{\dot{\alpha}_{y_\Delta}} \dot{\alpha}_{y_\Delta} + P_{\alpha_{y_\Delta}} \alpha_{y_\Delta} + P_{\dot{\alpha}_{x_\Delta}} \dot{\alpha}_{x_\Delta} \\
& + P_{\alpha_{x_\Delta}} \alpha_{x_\Delta} + P_{\ddot{x}} \ddot{x} + P_{\dot{x}} \dot{x} + P_{\dot{y}} \dot{y} \\
& + P_{\ddot{\beta}_{y_\Delta}} \ddot{\beta}_{y_\Delta} + P_{\dot{\beta}_{y_\Delta}} \dot{\beta}_{y_\Delta} + P_{\beta_{y_\Delta}} \beta_{y_\Delta} + P_{\dot{\beta}_{x_\Delta}} \dot{\beta}_{x_\Delta} \\
& + P_{\beta_{x_\Delta}} \beta_{x_\Delta} + P_{\ddot{\beta}_{c_\Delta}} \ddot{\beta}_{c_\Delta} + P_{\dot{\beta}_{c_\Delta}} \dot{\beta}_{c_\Delta} + P_{\beta_{c_\Delta}} \beta_{c_\Delta} \\
& + P_{\ddot{z}} \ddot{z} + P_{\dot{z}} \dot{z} + P_{\dot{\Omega}_\Delta} \dot{\Omega}_\Delta + P_{\Omega_\Delta} \Omega_\Delta \\
& = P_{\theta_{c_\Delta}} \theta_{c_\Delta} + P_{\theta_{y_\Delta}} \theta_{y_\Delta} + P_{\theta_{x_\Delta}} \theta_{x_\Delta}
\end{aligned}
\tag{71}
$$

The stability derivatives in equation (71) are defined by:

$$P_{\ddot{\alpha}_{y_\Delta}} = -I_y - h^2 b \frac{W_b}{g} + hb \frac{W_b}{g} x_s \alpha_{y_0}$$

$$
\begin{aligned}
P_{\dot{\alpha}_{y_\Delta}} = & -(h - x_s \alpha_{y_0}) h \Big[\beta_{c_s}^2 \Omega_0 \frac{A_2}{2} + \beta_{y_s}^2 \Omega_0 \frac{A_2}{2} \\
& + \beta_{y_s} \Omega_0 \frac{A_2}{2} (\theta_{y_s} + \alpha_{y_0}) + v \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2} \right) \\
& + \Omega_0 A_3 \Big] + (h \alpha_{y_0} + x_s) h \Big[-V_0 \left(\theta_{c_{s_R}} E_2 - \frac{\theta_d}{R} A_2 \right) \\
& + (\theta_{y_s} + \alpha_{y_0}) \Omega_0 A_2 \Big] + eh \Big[\alpha_{y_0} \beta_{c_s} \Omega_0 \frac{A_2}{8} (\theta_{y_s} \\
& + \alpha_{y_0}) + \beta_{c_s} \Omega_0 A_2 \left(\frac{1}{2} + \frac{7}{8} \alpha_{y_0} \beta_{y_s} \right) - \alpha_{y_0} \beta_{x_s} v \frac{E_2}{8}
\end{aligned}
$$

$$+\beta_{x_s} V_0 \frac{\epsilon_2}{4}\Big] - M_{Y_{T\alpha}} \frac{\bar{l}_T}{V_0}$$

$$P_{\alpha_{y_\Delta}} = -(h - \chi_s \alpha_{y_0})\Big[\Omega_0 V_0 \beta_{y_s} \frac{A_2}{2} - v\Omega_0 \frac{A_2}{2}\Big] - \chi_s\Big[-\beta_{c_s}^2 V_0 \Omega_0 \frac{A_2}{2}$$
$$-\beta_{c_s}\beta_{x_s}\Omega_0^2 \frac{C_2}{2} + \beta_{y_s}\Omega_0^2\Big(\theta_{cs_R} C_2 - \frac{\theta_d}{R} B_2\Big) - \Big(\beta_{y_s}\Omega_0 V_0 \frac{A_2}{2}$$
$$+ v\Omega_0 \frac{A_2}{2}\Big)(\theta_{y_s} + \alpha_{y_0}) - \beta_{y_s}^2 V_0 \Omega_0 \frac{A_2}{2} - \beta_{y_s}\Omega_0 v \frac{3A_2}{2}$$
$$- V_0 v\Big(\theta_{cs_R}\frac{\epsilon_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\Big) - \Omega_0 V_0 A_3\Big] + (h\alpha_{y_0} + \chi_s)\cdot$$
$$\cdot \Omega_0 V_0 A_2 + h\Big[bW_b - V_0^2\Big(\theta_{cs_R}\frac{\epsilon_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\Big) - \Omega_0^2\Big(\theta_{cs_R} C_2$$
$$- \frac{\theta_d}{R} B_2\Big) + (\theta_{y_s} + \alpha_{y_0})\Omega_0 V_0 A_2 + v\Omega_0 A_2\Big] + \Big[-\beta_{c_s}\Omega_0 V_0\cdot$$
$$\cdot \frac{A_2}{8}(\theta_{y_s} - 2\alpha_{y_0}) - \Omega_0^2 \frac{L_4}{2} + \beta_{c_s}\Omega_0^2\Big(\theta_{cs_R}\frac{C_2}{2} - \frac{\theta_d}{R}\frac{B_2}{2}\Big)$$
$$-\beta_{c_s}\Omega_0 A_2\Big(\beta_{y_s} V_0 \frac{7}{8} + \frac{1}{2}v\Big) - \beta_{x_s}\Big(\beta_{y_s}\Omega_0^2\frac{C_2}{2} - vV_0\frac{\epsilon_2}{8}\Big)\Big]$$
$$+ C_{m_{f\alpha}}\,\rho\frac{V_0^2}{2}\pi R^3 - M_{y_{T\alpha}}$$

$$P_{\dot{\alpha}_{x_\Delta}} = h(h - \chi_s\alpha_{y_0})\Big[-\beta_{c_s}\beta_{y_s}V_0\epsilon_2 - (\theta_{y_s} + \alpha_{y_0})\Big(\beta_{c_s}V_0\frac{\epsilon_2}{2}$$
$$+ \beta_{x_s}\Omega_0\frac{A_2}{2}\Big) + \beta_{x_s}\beta_{y_s}\Omega_0\frac{A_2}{2} - \beta_{c_s}v\frac{3}{2}\epsilon_2 + \beta_{c_s}\Omega_0\Big(\theta_{cs_R}\frac{3A_2}{2}$$
$$- \frac{\theta_d}{R}\frac{3C_2}{2}\Big) + \beta_{x_s}V_0\Big(\theta_{cs_R}\frac{\epsilon_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\Big)\Big] + h\Big[\alpha_{y_0}\beta_{c_s}\cdot$$
$$\cdot\Big(\beta_{c_s}V_0\frac{\epsilon_2}{2} + \beta_{x_s}\Omega_0\frac{3A_2}{8}\Big) + \Big(\alpha_{y_0}\beta_{y_s}V_0\frac{\epsilon_2}{8} - \alpha_{y_0}v\frac{\epsilon_2}{8}\Big)(\theta_{y_s} + \alpha_{y_0})$$
$$+ V_0\frac{\epsilon_2}{4}(\theta_{y_s} + \alpha_{y_0}) - \Omega_0(\alpha_{y_0}\beta_{y_s} + 1)\Big(\theta_{cs_R}A_2 - \frac{\theta_d}{R}C_2\Big) + (5v$$
$$+ 3\beta_{y_s}V_0)\alpha_{y_0}\beta_{y_s}\frac{\epsilon_2}{8} + \alpha_{y_0}V_0\Big(\beta_{x_s}^2\frac{\epsilon_2}{8} + \frac{\epsilon_3}{4}\Big) + \frac{\epsilon_2}{2}\Big(v$$
$$+ \beta_{y_s}\frac{V_0}{2}\Big)\Big]$$

$$P_{\alpha_{x_\Delta}} = -(h - \chi_s\alpha_{y_0})\Big[\beta_{c_s}\Omega_0^2\frac{C_2}{2}\Big] + \Big[\alpha_{y_0}\Big(\beta_{y_s}\Omega_0^2\frac{C_2}{2} - V_0 v\frac{\epsilon_2}{8}\Big)$$
$$+ V_0^2\frac{\epsilon_2}{8} + \Omega_0^2\frac{C_2}{2} - \beta_{c_s}\beta_{x_s}V_0\Omega_0\frac{A_2}{8} + (\beta_{y_s}\Omega_0 V_0 - \Omega_0 v)\cdot$$
$$\cdot\Big(\theta_{cs_R}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\Big) - \Big(\beta_{y_s}\Omega_0^2\frac{C_2}{2} - V_0 v\frac{3\epsilon_2}{8}\Big)(\theta_{y_s} +$$

$$+ \alpha_{y_0}) + \beta_{y_s} V_0 v \frac{E_2}{8} + \beta_{y_s}^2 \Omega_0^2 \frac{C_2}{2} + v^2 \frac{E_2}{2} - V_0^2 \frac{3E_3}{8}$$
$$- \Omega_0^2 C_3/2$$

$$P_{\ddot{x}} = -(h - x_s \alpha_{y_0})\, b \frac{W_b}{g}$$

$$P_{\dot{x}} = (P_{\dot{\alpha}_{y_\Delta}}/h) + M_{Y_{T\alpha}} \frac{\bar{l}_T}{V_0} h$$

$$P_{\ddot{y}} = P_{\dot{\alpha}_{x_\Delta}}/h$$

$$P_{\ddot{\beta}_{y_\Delta}} = \frac{L_4}{2}$$

$$P_{\dot{\beta}_{y_\Delta}} = (h - x_s \alpha_{y_0})\left[-\beta_{c_s} \Omega_0 \frac{C_2}{2} + \beta_{x_s} V_0 \frac{A_2}{8}\right] + \left[\alpha_{y_0} \beta_{y_s} \Omega_0 \frac{5C_2}{8}\right.$$
$$+ \alpha_{y_0} V_0 \left(\theta_{c_{s_R}} \frac{A_2}{8} - \frac{\theta_d}{R} \frac{C_2}{8}\right) + \Omega_0 \frac{C_2}{2} - \alpha_{y_0} \Omega_0 \frac{C_2}{8} (\theta_{y_s}$$
$$\left. + \alpha_{y_0})\right]$$

$$P_{\beta_{y_\Delta}} = (h - x_s \alpha_{y_0})\left[-\beta_{y_s} V_0 \Omega_0 A_2 + \Omega_0^2 \left(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2\right)\right.$$
$$\left. - V_0 \Omega_0 \frac{A_2}{2} (\theta_{y_s} + \alpha_{y_0}) - \Omega_0 v \frac{3A_2}{2}\right]$$
$$+ \alpha_{y_0} \Omega_0 \left(\beta_{c_s} V_0 \frac{7}{8} A_2 + \beta_{x_s} \Omega_0 \frac{C_2}{2}\right) - \Omega_0^2 \frac{L_4}{2}$$

$$P_{\dot{\beta}_{x_\Delta}} = (h - x_s \alpha_{y_0})\left[\beta_{y_s} V_0 \frac{A_2}{8} - \Omega_0 \left(\theta_{c_{s_R}} \frac{C_2}{2} - \frac{\theta_d}{R} \frac{B_2}{2}\right) + v A_2\right.$$
$$\left. + V_0 \frac{3A_2}{8} (\theta_{y_s} + \alpha_{y_0})\right] + (h \alpha_{y_0} + x_s) V_0 \frac{A_2}{2}$$
$$+ \left[2 \Omega_0 \frac{L_4}{2} - \frac{1}{8} \alpha_{y_0} (\beta_{c_s} V_0 A_2 + \beta_{x_s} \Omega_0 C_2)\right]$$

$$P_{\beta_{x_\Delta}} = -(h - x_s \alpha_{y_0}) \beta_{c_s} \Omega_0^2 \frac{C_2}{2} + \left[\alpha_{y_0} \beta_{y_s} \Omega_0^2 \frac{C_2}{2}\right.$$
$$\left. - V_0 \frac{E_2}{8} (\alpha_{y_0} v V_0 + V_0) + \Omega_0^2 C_2/2\right]$$

$$P_{\ddot{\beta}_{c_\Delta}} = (h \alpha_{y_0} + x_s)\, b J_b$$

$$P_{\dot{\beta}_{c_\Delta}} = (h - x_s \alpha_{y_0})\Big[-\beta_{y_s}\Omega_0 \frac{3C_2}{2} - V_0\Big(\theta_{c_{s_R}}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\Big)$$
$$+ \Omega_0 \frac{C_2}{2}(\theta_{y_s} + \alpha_{y_0})\Big] + (h\alpha_{y_0} + x_s)\Omega_0 C_2$$
$$+ \Big[\alpha_{y_0}\Big(\beta_{c_s}\Omega_0 \frac{C_2}{2} - \beta_{x_s} V_0 \frac{A_2}{8}\Big)\Big]$$

$$P_{\beta_{c_\Delta}} = (h - x_s\alpha_{y_0})\Big[-\beta_{c_s}V_0\Omega_0 A_2 - \beta_{x_s}\Omega_0^2\frac{C_2}{2}\Big] + \Big[-\alpha_{y_0}\Omega_0^2\Big(\theta_{c_{s_R}}\frac{C_2}{2}$$
$$- \frac{\theta_d}{R}\frac{B_2}{2}\Big) + \alpha_{y_0}\Omega_0 V_0 \frac{A_2}{8}(\theta_{y_s} + \alpha_{y_0} + 7\beta_{y_s}) + \Omega_0 \frac{A_2}{2}\cdot$$
$$\cdot(\alpha_{y_0} v + V_0)$$

$$P_{\ddot{z}} = -(h\alpha_{y_0} + x_s)\, b\, \frac{W_b}{g}$$

$$P_{\dot{z}} = (h - x_s\alpha_{y_0})\Big[V_0\Big(\theta_{c_{s_R}}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\Big) - \Omega_0\frac{A_2}{2}(\theta_{y_s} + \alpha_{y_0})$$
$$+ \beta_{y_s}\Omega_0\frac{3A_2}{2}\Big] - (h\alpha_{y_0} + x_s)\Omega_0 A_2 + \Big[-\alpha_{y_0}\beta_{c_s}\Omega_0\frac{A_2}{2}$$
$$+ \alpha_{y_0}\beta_{x_s}V_0\frac{E_2}{8}\Big] + M_{Y_{T_\alpha}}/V_0$$

$$P_{\dot{\Omega}_\Delta} = \beta_{x_s}\frac{L_4}{2}$$

$$P_{\Omega_\Delta} = (h - x_s\alpha_{y_0})\Big[-\beta_{c_s}\beta_{x_s}\Omega_0 C_2 + \beta_{y_s}\Omega_0\Big(\theta_{c_{s_R}} 2C_2$$
$$- \frac{\theta_d}{R} 2B_2\Big) - (\theta_{y_s} + \alpha_{y_0})\Big(\beta_{y_s}V_0\frac{A_2}{2} - v\frac{A_2}{2}\Big) - \beta_{c_s}^2 V_0 \frac{A_2}{2}$$
$$- \beta_{y_s}^2 V_0\frac{A_2}{2} - \beta_{y_s} v \frac{3}{2}A_2 - V_0 A_3\Big] + (h\alpha_{y_0} + x_s)\cdot$$
$$\cdot\Big[-2\Omega_0\Big(\theta_{c_{s_R}}C_2 - \frac{\theta_d}{R}B_2\Big) + (\theta_{y_s} + \alpha_{y_0})V_0 A_2$$
$$+ v A_2\Big] + \Big[\alpha_{y_0}\Omega_0 L_4 - \alpha_{y_0}\beta_{c_s}\Omega_0\Big(\theta_{c_{s_R}}C_2 - \frac{\theta_d}{R}B_2\Big)$$
$$+ \alpha_{y_0}\beta_{c_s}V_0\frac{A_2}{8}(\theta_{y_s} + \alpha_{y_0}) + \alpha_{y_0}\beta_{c_s}\frac{A_2}{2}\Big(\beta_{y_s}V_0\frac{7}{4}$$
$$+ v\Big) - \beta_{y_s}\Omega_0(L_4 - \alpha_{y_0}\beta_{x_s}C_2) + \beta_{x_s}\Omega_0 C_2$$
$$+ \beta_{c_s} V_0 \frac{A_2}{2}\Big]$$

$$P_{\theta_{c_\Delta}} = -(h - x_s \alpha_{y_0})\left[\beta_{y_s} \Omega_0^2 C_2 - V_0 v \frac{E_2}{2}\right] + (h \alpha_{y_0} + x_s)\left[+V_0^2 \frac{E_2}{2} + \Omega_0^2 C_2\right] + \alpha_{y_0} \beta_{c_s} \Omega_0^2 \frac{C_2}{2}$$

$$P_{\theta_{y_\Delta}} = -(h - x_s \alpha_{y_0})\left[\Omega_0 V_0 \beta_{y_s} \frac{A_2}{2} - v \Omega_0 \frac{A_2}{2}\right] + (h \alpha_{y_0} + x_s) \Omega_0 V_0 A_2 + \alpha_{y_0} \beta_{c_s} \Omega_0 V_0 \frac{A_2}{8}$$

$$P_{\theta_{x_\Delta}} = -(h - x_s \alpha_{y_0}) \beta_{c_s} \Omega_0^2 \frac{C_2}{2} + \alpha_{y_0}\left(\beta_{y_s} \Omega_0^2 \frac{C_2}{2} - V_0 v \frac{E_2}{8}\right) - V_0^2 \frac{E_2}{8} + \Omega_0^2 \frac{C_2}{2}$$

4.5 THE EQUATION OF HELICOPTER ROLLING MOTION

Figure 10 following shows the external forces and moments which influence the rolling motion of the helicopter.

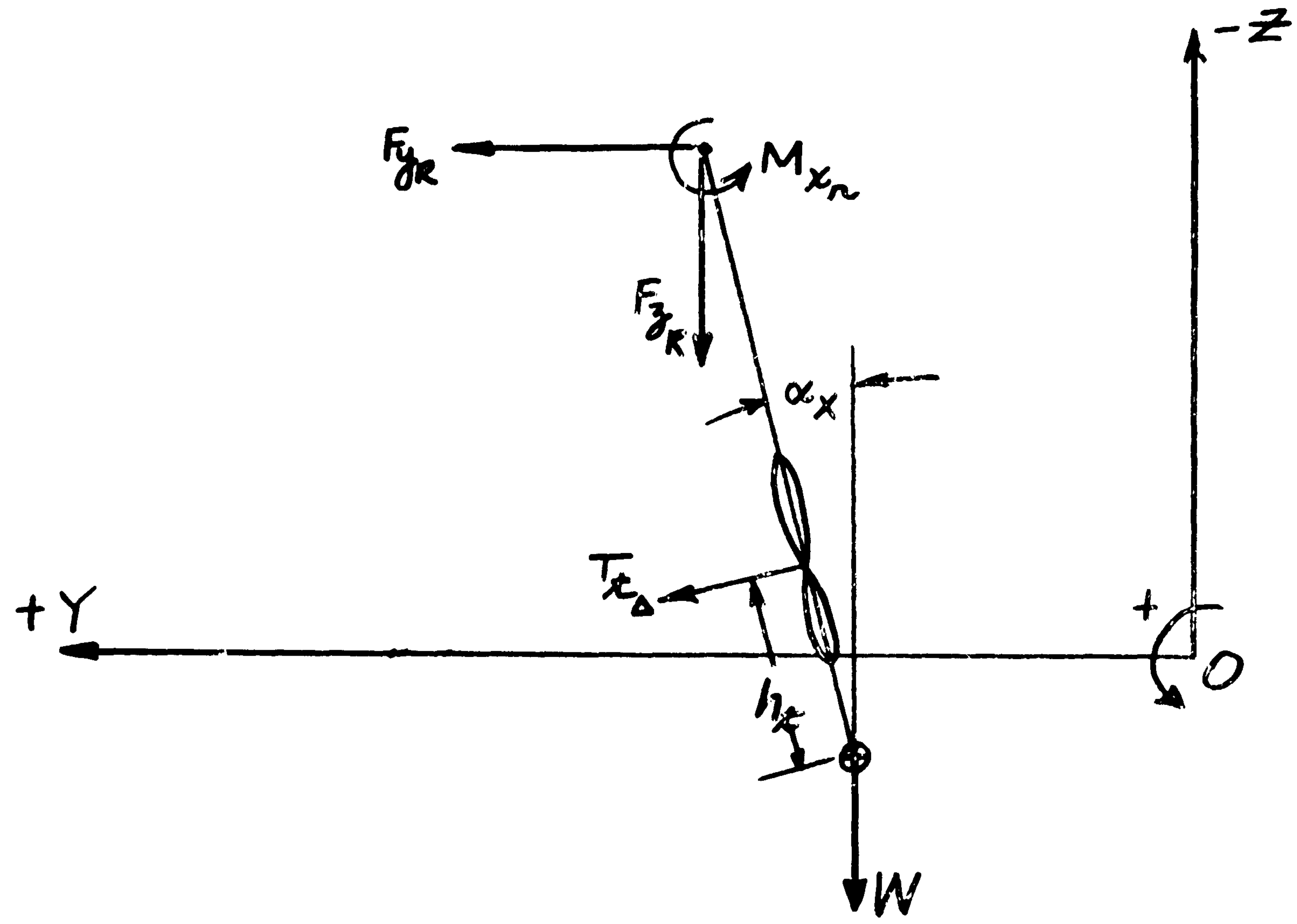

Figure 10

Lateral Forces and Moments

From the figure,

$$F_{y_R} h + F_{z_R} h \alpha_{x_\Delta} + M_{x_r} + T_{t_\Delta} h_t = I_x \ddot{\alpha}_{x_\Delta} \qquad (72)$$

The terms required for equation (72) are given by equations (38), (39), (47), and (57). Expanding the equation into the desired form yields

$$R_{\ddot{\alpha}_{x_\Delta}} \ddot{\alpha}_{x_\Delta} + R_{\dot{\alpha}_{x_\Delta}} \dot{\alpha}_{x_\Delta} + R_{\alpha_{x_\Delta}} \alpha_{x_\Delta} + R_{\dot{\alpha}_{y_\Delta}} \dot{\alpha}_{y_\Delta}$$
$$+ R_{\alpha_{y_\Delta}} \alpha_{y_\Delta} + R_{\ddot{y}} \ddot{y} + R_{\dot{y}} \dot{y} + R_{\ddot{\beta}_{x_\Delta}} \ddot{\beta}_{x_\Delta}$$
$$+ R_{\dot{\beta}_{x_\Delta}} \dot{\beta}_{x_\Delta} + R_{\beta_{x_\Delta}} \beta_{x_\Delta} + R_{\dot{\beta}_{y_\Delta}} \dot{\beta}_{y_\Delta} + R_{\beta_{y_\Delta}} \beta_{y_\Delta} +$$

$$+ R_{\dot{\beta}_{c_\Delta}} \dot{\beta}_{c_\Delta} + R_{\beta_{c_\Delta}} \beta_{c_\Delta} + R_{\dot{x}} \dot{x} + R_{\dot{z}} \dot{z} + R_{\dot{\alpha}_{z_\Delta}} \dot{\alpha}_{z_\Delta}$$

$$+ R_{\alpha_{z_\Delta}} \alpha_{z_\Delta} + R_{\dot{\Omega}_\Delta} \dot{\Omega}_\Delta + R_{\Omega_\Delta} \Omega_\Delta = R_{\theta_{c_\Delta}} \theta_{c_\Delta}$$

$$+ R_{\theta_{x_\Delta}} \theta_{x_\Delta} + R_{\theta_{y_\Delta}} \theta_{y_\Delta} + R_{\theta_{\Delta t}} \theta_{\Delta t} \qquad (73)$$

The stability derivatives in equation (73) are defined by:

$$R_{\ddot{\alpha}_{x_\Delta}} = -h^2 b \frac{W_b}{g} - I_x$$

$$R_{\dot{\alpha}_{x_\Delta}} = -h^2 \Big[\beta_{c_s} \beta_{x_s} V_0 E_2 - \beta_{c_s}^2 \Omega_0 \frac{A_2}{2} - \beta_{x_s}^2 \Omega_0 \frac{A_2}{2}$$

$$-(\beta_{y_s} V_0 + v)\left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) - \Omega_0 A_3\Big]$$

$$+ h\Big[-\alpha_{y_0} \beta_{c_s} V_0 \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) + (\theta_{y_s} + \alpha_{y_0}) \cdot$$

$$\cdot\, \alpha_{y_0} \left(\beta_{c_s} \Omega_0 \frac{A_2}{8} + \beta_{x_s} V_0 \frac{E_2}{8}\right) - \alpha_{y_0} \beta_{y_s} \Big(\beta_{c_s} \Omega_0 \frac{A_2}{8}$$

$$+ \beta_{x_s} V_0 \frac{E_2}{4}\Big) - \alpha_{y_0} \beta_{x_s} \Omega_0 \left(\theta_{c_{s_R}} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2}\right)$$

$$- \alpha_{y_0} \beta_{x_s} v \frac{E_2}{8} - \beta_{x_s} V_0 \frac{E_2}{4} - \beta_{c_s} \Omega_0 \frac{A_2}{2}\Big]$$

$$R_{\alpha_{x_\Delta}} = h\left[\beta_{y_s} V_0 + v\right] \Omega_0 \frac{A_2}{2} + h\Big[-\alpha_{y_0} \beta_{c_s} V_0 \Big(\theta_{c_{s_R}} \frac{E_2}{2}$$

$$- \frac{\theta_d}{R}\frac{A_2}{2}\Big) + (\theta_{y_s} + \alpha_{y_0}) \alpha_{y_0} \Big(\beta_{c_s} \Omega_0 \frac{A_2}{8}$$

$$+ \beta_{x_s} V_0 \frac{E_2}{8}\Big) - \alpha_{y_0} \beta_{y_s} \left(\beta_{c_s} \Omega_0 \frac{A_2}{8} + \beta_{x_s} V_0 \frac{E_2}{4}\right)$$

$$- \alpha_{y_0} \beta_{x_s} \Omega_0 \left(\theta_{c_{s_R}} \frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right) - \alpha_{y_0} \beta_{x_s} v \frac{E_2}{8}$$

$$- \beta_{x_s} V_0 \frac{E_2}{4} - \beta_{c_s} \Omega_0 \frac{A_2}{2}\Big] + h\Big[b W_b - V_0^2 \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right)$$

$$- \Omega_0^2 \left(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2\right) + (\theta_{y_s} + \alpha_{y_0}) \Omega_0 V_0 A_2 + v \Omega_0 A_2\Big]$$

$$R_{\dot{\alpha}y_\Delta} = h^2\Big[(\theta_{y_s} + \alpha_{y_0})(\beta_{c_s} V_0 E_2 + \beta_{x_s}\Omega_0 A_2) - 3\beta_{c_s}\Omega_0\left(\theta_{c_{s_R}}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right) - \beta_{x_s}V_0\left(\theta_{c_{s_R}}E_2 - \frac{\theta_d}{R}A_2\right) + \beta_{c_s}\beta_{y_s}V_0 2E_2 + \beta_{x_s}\beta_{y_s}\Omega_0\frac{A_2}{2} + \beta_{c_s}v\frac{3}{2}E_2\Big] + eh\Big[\alpha_{y_0}\beta_{c_s}(\beta_{c_s}V_0E_2 + \beta_{x_s}\Omega_0\frac{9}{8}A_2 + \alpha_{y_0}\beta_{x_s}^2 V_0\frac{E_2}{4} - \alpha_{y_0}\beta_{y_s}\Omega_0\left(\theta_{c_{s_R}}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right) + (\theta_{y_s} + \alpha_{y_0})\frac{E_2}{4}\cdot \cdot(\alpha_{y_0}\beta_{y_s}V_0 + \alpha_{y_0}v/2 + V_0 3) + \alpha_{y_0}\beta_{y_s}\frac{E_2}{4}(3\beta_{y_s}V_0 + \frac{7}{2}v) - \alpha_{y_0}V_0\frac{E_3}{4} - \Omega_0\left(\theta_{c_{s_R}}A_2 - \frac{\theta_d}{R}C_2\right) + \left(\beta_{y_s}\frac{V_0}{2} + v\right)\frac{E_2}{2}\Big]$$

$$R_{\alpha y_\Delta} = h\Big[\beta_{c_s}\left(V_0^2\frac{E_2}{2} + \Omega_0^2\frac{C_2}{2}\right) + \beta_{x_s}V_0\Omega_0 A_2\Big] + \Big[V_0^2\frac{3E_2}{8} + \alpha_{y_0}V_0\frac{E_2}{8}(\beta_{y_s}V_0 + v) + \Omega_0^2\frac{C_2}{2} + \beta_{c_s}V_0\left(\beta_{c_s}V_0\frac{E_2}{2} + \beta_{x_s}\Omega_0\frac{9}{8}A_2\right) + \beta_{x_s}^2\left(V_0^2\frac{E_2}{8} + \Omega_0^2\frac{C_2}{8}\right) - \left(\theta_{c_{s_R}}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right)(\beta_{y_s}V_0\Omega_0 + v\Omega_0) + (\theta_{y_s} + \alpha_{y_0})(\beta_{y_s}V_0 + v)V_0\frac{E_2}{8} + \beta_{y_s}V_0\frac{E_2}{8}(7v + 3\beta_{y_s}V_0) + v^2\frac{E_2}{2} - V_0^2\frac{E_3}{8} - \Omega_0^2\frac{C_3}{2}\Big]$$

$$R_{\ddot{y}} = -hb\frac{W_b}{g}$$

$$R_{\dot{y}} = R_{\dot{\alpha}x_\Delta}/h - h_t\Omega_{\alpha t}A_4$$

$$R_{\ddot{\beta}x_\Delta} = L_4/2$$

$$R_{\dot{\beta}x_\Delta} = h\Big[\beta_{c_s}\Omega_0\frac{C_2}{2} + \beta_{x_s}V_0\frac{5}{8}A_2\Big] + \Big[\Omega_0\frac{C_2}{2}\left(1 - \alpha_{y_0}\frac{\beta_{y_s}}{4}\right) - \alpha_{y_0}V_0\left(\theta_{c_{s_R}}\frac{A_2}{8} - \frac{\theta_d}{R}\frac{C_2}{8}\right) + \alpha_{y_0}\Omega_0\frac{C_2}{8}(\theta_{y_s} + \alpha_{y_0})\Big]$$

$$R_{\beta x_\Delta} = h\Big[-V_0^2\Big(\theta_{cs_R}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\Big) - \Omega_0^2\Big(\theta_{cs_R}C_2 - \frac{\theta_d}{R}B_2\Big)$$
$$+ V_0\Omega_0 A_2(\theta_{y_s} + \alpha_{y_0}) + \beta_{y_s}V_0\Omega_0\frac{A_2}{2} + v\Omega_0\frac{3}{2}A_2\Big]$$
$$- \Omega_0^2\frac{L_4}{2} + \alpha_{y_0}\beta_{c_s}V_0\Omega_0\frac{9}{8}A_2 + \alpha_{y_0}\beta_{x_s}\Big(V_0^2\frac{E_2}{4} + \Omega_0^2C_2\Big)\Big]$$

$$R_{\dot\beta y_\Delta} = h\Big[\beta_{y_s}V_0\frac{7A_2}{8} - \Omega_0\Big(\theta_{cs_R}\frac{C_2}{2} - \frac{\theta_d}{R}\frac{B_2}{2}\Big) + vA_2$$
$$+ V_0\frac{A_2}{8}(\theta_{y_s} + \alpha_{y_0})\Big] + \Big[\alpha_{y_0}\beta_{c_s}V_0\frac{78}{A_2} + \alpha_{y_0}\beta_{x_s}\Omega_0\frac{7}{8}C_2$$
$$- \Omega_0 L_4\Big]$$

$$R_{\beta y_\Delta} = h\Big[\beta_{c_s}V_0^2E_2 - \beta_{c_s}\Omega_0^2\frac{C_2}{2} + \beta_{x_s}V_0\Omega_0\frac{A_2}{2}\Big] + \Big[\alpha_{y_0}V_0\frac{E_2}{4}\Big(v\frac{7}{2}$$
$$+ 3\beta_{y_s}V_0\Big) - \alpha_{y_0}V_0\Omega_0\Big(\theta_{cs_R}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\Big) + \alpha_{y_0}\Omega_0^2\frac{C_2}{4}(\theta_{y_s}$$
$$+ \alpha_{y_0}) + V_0^2\frac{E_2}{8} - \Omega_0^2\frac{C_2}{2}\Big]$$

$$R_{\dot\beta c_\Delta} = h\Big[\frac{3}{2}(\beta_{c_s}V_0A_2 + \beta_{x_s}\Omega_0C_2)\Big] + \Big[\alpha_{y_0}vA_2 + V_0\frac{A_2}{2} + \alpha_{y_0}\beta_{y_s}V_0\frac{7}{8}A_2$$
$$- \alpha_{y_0}\Omega_0\Big(\theta_{cs_R}\frac{C_2}{2} - \frac{\theta_d}{R}\frac{B_2}{2}\Big) - \alpha_{y_0}V_0\frac{A_2}{8}(\theta_{y_s} + \alpha_{y_0})\Big]$$

$$R_{\beta c_\Delta} = h\Big[(\theta_{y_s} + \alpha_{y_0})\Big(V_0^2\frac{E_2}{2} + \Omega_0^2\frac{C_2}{2}\Big) - 3V_0\Omega_0\Big(\theta_{cs_R}\frac{A_2}{2}$$
$$- \frac{\theta_d}{R}\frac{C_2}{2}\Big) + \beta_{y_s}V_0^2E_2 + V_0v\frac{3E_2}{2} - \beta_{y_s}\Omega_0^2\frac{C_2}{2}\Big] + \Big[\alpha_{y_0}V_0\,\cdot$$
$$\cdot\Big(\beta_{c_s}V_0E_2 + \Omega_0\beta_{x_s}\frac{9}{8}A_2\Big)\Big]$$

$$R_{\dot z} = (R_{\dot\alpha y_\Delta}/h) + h_t V_0 E_4\,\theta_{0t}$$

$$R_{\dot z} = h\Big[-\beta_{c_s}V_0\frac{3}{2}E_2 - \beta_{x_s}\Omega_0\frac{3}{2}A_2\Big] + \Big[-\alpha_{y_0}V_0\frac{E_2}{8}(7\beta_{y_s}$$
$$+ \theta_{y_s} + \alpha_{y_0}) + \alpha_{y_0}\Omega_0\Big(\theta_{cs_R}\frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\Big) - V_0\frac{E_2}{2}$$
$$- \alpha_{y_0}vE_2\Big]$$

$$R_{\dot{\alpha}_{z_\Delta}} = +h_t \Omega_{o_t} l_t A_4$$

$$R_{\alpha_{z_\Delta}} = h_t \Omega_{o_t} V_o A_4$$

$$\begin{aligned}R_{\Omega_\Delta} = {} & h\Big[(\theta_{y_s} + \alpha_{y_o})(\beta_{c_s}\Omega_o C_2 + \beta_{x_s} V_o A_2) - 3\beta_{c_s} V_o\Big(\theta_{c_{s_R}}\frac{A_2}{2} \\ & - \frac{\theta_d}{R}\frac{C_2}{2}\Big) - 2\beta_{x_s}\Omega_o\Big(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2\Big) - \beta_{c_s}\beta_{y_s}\Omega_o C_2 \\ & + \beta_{x_s}\Big(\vartheta \frac{3}{2} A_2 + \beta_{y_s} V_o \frac{A_2}{2}\Big)\Big] + \Big[\alpha_{y_o}\beta_{c_s}\beta_{x_s} V_o \frac{9}{8} A_2 \\ & + \alpha_{y_o}\beta_{x_s}^2 \Omega_o C_2 - \alpha_{y_o}(\beta_{y_s} V_o + \vartheta)\Big(\theta_{c_{s_R}}\frac{A_2}{2} \\ & - \frac{\theta_d}{R}\frac{C_2}{2}\Big) - \beta_{x_s}\Omega_o L_4 - V_o\Big(\theta_{c_{s_R}} A_2 - \frac{\theta_d}{R} C_2\Big) \\ & + \Omega_o C_2(\theta_{y_s} + \alpha_{y_o}) - \beta_{y_s}\Omega_o C_2 - \alpha_{y_o}\Omega_o C_3\Big] \\ & + h_t \frac{\Omega_o}{\Omega_{o_t}}\Big[\theta_{o_t} 2\Omega_{o_t} C_4 - \vartheta_t A_4\Big]\end{aligned}$$

$$R_{\dot{\Omega}_\Delta} = +\frac{L_4}{2}(\alpha_{y_o} + \beta_{y_s})$$

$$\begin{aligned}R_{\theta_{c_\Delta}} = {} & -h\Big[-\beta_{x_s}\Big(V_o^2\frac{E_2}{2} + \Omega_o^2 C_2\Big) - 3\beta_{c_s} V_o \Omega_o \frac{A_2}{2}\Big] \\ & + \Big[\alpha_{y_o}\Omega_o \frac{A_2}{2}(\beta_{y_s} V_o + \vartheta) + \Omega_o V_o A_2\Big]\end{aligned}$$

$$\begin{aligned}R_{\theta_{x_\Delta}} = {} & h\Big[+\beta_{y_s} V_o \Omega_o \frac{A_2}{2} + \Omega_o \vartheta \frac{A_2}{2}\Big] + \Big[\alpha_{y_o}\beta_{c_s}\Omega_o V_o \frac{5}{8} A_2 \\ & + \alpha_{y_o}\beta_{x_s}\Big(V_o^2\frac{E_2}{8} + \Omega_o^2\frac{C_2}{2}\Big)\Big]\end{aligned}$$

$$\begin{aligned}R_{\theta_{y_\Delta}} = {} & h\Big[\beta_{c_s}\Big(V_o^2\frac{E_2}{2} + \Omega_o^2\frac{C_2}{2}\Big) + \beta_{x_s} V_o \Omega_o A_2\Big] + \Big[\alpha_{y_o} V_o \frac{E_2}{8} \cdot \\ & \cdot(\beta_{y_s} V_o + \vartheta) + V_o^2 \frac{3}{8} E_2 + \Omega_o^2 \frac{C_2}{2}\Big]\end{aligned}$$

$$R_{\theta_{\Delta_t}} = -\Omega_{o_t}^2 C_4 - V_o^2 E_4/2$$

4.6 THE EQUATION OF HELICOPTER YAWING MOTION

The forces and moments acting on the "free-body" helicopter which tend to yaw the aircraft are shown in Figure 11.

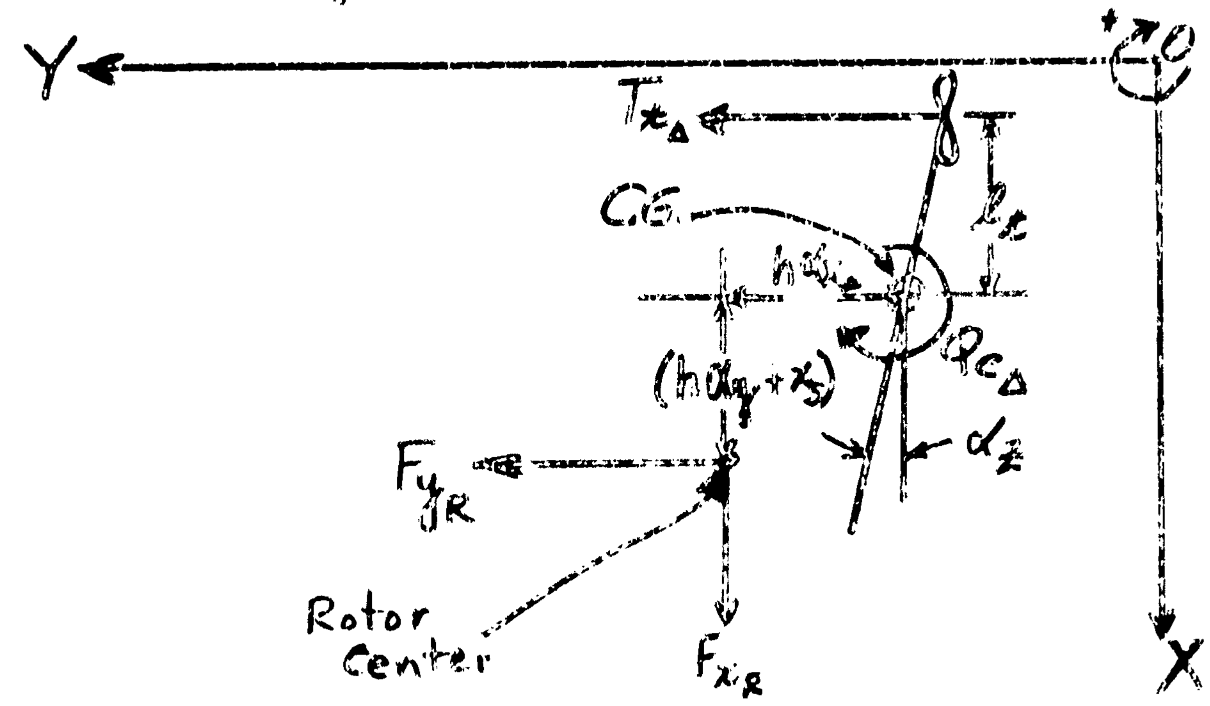

Figure 11

Yawing Forces and Moments

From the figure,

$$Q_{e_\Delta} + F_{y_R}(h\alpha_y + x_3) - T_{t_\Delta} l_t - F_{x_R} h \alpha_{x_\Delta} = I_z \ddot{\alpha}_{z_\Delta} \qquad (74)$$

In stability derivative form this is

$$\begin{aligned} & S_{\ddot\alpha_{z_\Delta}}\ddot\alpha_{z_\Delta} + S_{\dot\alpha_{z_\Delta}}\dot\alpha_{z_\Delta} + S_{\alpha_{z_\Delta}}\alpha_{z_\Delta} + S_{\ddot\alpha_{x_\Delta}}\ddot\alpha_{x_\Delta} \\ & + S_{\dot\alpha_{x_\Delta}}\dot\alpha_{x_\Delta} + S_{\alpha_{x_\Delta}}\alpha_{x_\Delta} + S_{\dot\alpha_{y_\Delta}}\dot\alpha_{y_\Delta} + S_{\alpha_{y_\Delta}}\alpha_{y_\Delta} \\ & + S_{\dot\beta_{c_\Delta}}\dot\beta_{c_\Delta} + S_{\beta_{c_\Delta}}\beta_{c_\Delta} + S_{\dot\beta_{x_\Delta}}\dot\beta_{x_\Delta} + S_{\beta_{x_\Delta}}\beta_{x_\Delta} + S_{\dot\beta_{y_\Delta}}\dot\beta_{y_\Delta} + S_{\beta_{y_\Delta}}\beta_{y_\Delta} \\ & + S_{\dot x}\dot x + S_{\ddot y}\ddot y + S_{\dot y}\dot y + S_{\dot z}\dot z + S_{\Omega_\Delta}\Omega_\Delta \\ & = Q_{R_\Delta} + S_{\theta_{c_\Delta}}\theta_{c_\Delta} + S_{\theta_{y_\Delta}}\theta_{y_\Delta} + S_{\theta_{x_\Delta}}\theta_{x_\Delta} + S_{\theta_{\Delta t}} \end{aligned} \qquad (75)$$

The stability derivatives are given by

$$S_{\ddot\alpha_{z_\Delta}} = -I_z$$

$$S_{\ddot{\alpha}z_\Delta} = -l_t^2 \Omega_{\alpha_t} A_4$$

$$S_{\alpha z_\Delta} = -l_t V_0 A_4 \Omega_{\alpha_t}$$

$$S_{\ddot{\alpha}x_\Delta} = -(h\alpha_{y_0} + x_s) h b \frac{W_b}{g}$$

$$S_{\dot{\alpha}x_\Delta} = -(h\alpha_{y_0} + x_s) h \Big[\beta_{c_s}\beta_{x_s} V_0 E_2 + \beta_{c_s}^2 \Omega_0 \frac{A_2}{2} + \beta_{x_s}^2 \Omega_0 \frac{A_2}{2}$$
$$+ \beta_{y_s} V_0 \left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) + v\left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right)$$
$$+ \Omega_0 A_3\Big]$$

$$S_{\alpha x_\Delta} = (h\alpha_{y_0} + x_s)\left[\beta_{y_s} V_0 \Omega_0 \frac{A_2}{2} + \Omega_0 v \frac{A_2}{2}\right] - h \cdot$$
$$\cdot \Big[-\beta_{c_s}^2 V_0 \Omega_0 \frac{A_2}{2} - \beta_{c_s}\beta_{x_s} \Omega_0^2 \frac{C_2}{2} + \beta_{y_s}\Omega_0^2\left(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2\right)$$
$$- \beta_{y_s}\Omega_0 V_0 \frac{A_2}{2}(\theta_{y_s} + \alpha_{y_0}) - \beta_{y_s}^2 V_0 \Omega_0 \frac{A_2}{2} - \beta_{y_s}\Omega_0 v \frac{3A_2}{2}$$
$$- V_0 v\left(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) + v\Omega_0 \frac{A_2}{2}(\theta_{y_s} + \alpha_{y_0})$$
$$- \Omega_0 V_0 A_3\Big]$$

$$S_{\dot{\alpha}y_\Delta} = (h\alpha_{y_0} + x_s) h \Big[(\theta_{y_s} + \alpha_{y_0})(\beta_{c_s} V_0 E_2 + \beta_{x_s}\Omega_0 A_2)$$
$$- 3\beta_{c_s}\Omega_0\left(\theta_{c_{s_R}} \frac{A_2}{2} - \frac{\theta_d}{R}\frac{C_2}{2}\right) - \beta_{x_s} V_0\left(\theta_{c_{s_R}} E_2 - \frac{\theta_d}{R} A_2\right)$$
$$+ \beta_{c_s}\beta_{y_s} V_0 2 E_2 + \beta_{x_s}\beta_{y_s}\Omega_0 \frac{A_2}{2} + \left(\beta_{c_s} v \frac{3}{2} E_2\right)\Big]$$

$$S_{\alpha y_\Delta} = (h\alpha_{y_0} + x_s)\left[\beta_{c_s}\left(V_0^2 \frac{E_2}{2} + \Omega_0^2 \frac{C_2}{2}\right) + \beta_{x_s} V_0 \Omega_0 A_2\right]$$
$$+ h\Big[\beta_{c_s}\beta_{y_s} V_0^2 E_2 + V_0 v \beta_{c_s} \frac{3}{2} E_2 - \beta_{c_s}\beta_{y_s}\Omega_0^2 \frac{C_2}{2}$$
$$+ \beta_{c_s} V_0^2 \frac{E_2}{2}(\theta_{y_s} + \alpha_{y_0}) + \beta_{c_s}\Omega_0^2 \frac{C_2}{2}(\theta_{y_s} + \alpha_{y_0})$$
$$+ \beta_{x_s}\beta_{y_s} V_0 \Omega_0 \frac{A_2}{2} + \beta_{x_s} v \Omega_0 \frac{3}{2} A_2 - \beta_{x_s} V_0^2 \Big(\theta_{c_{s_R}} \frac{E_2}{2}$$

$$- \frac{\theta_d}{R} \frac{A_2}{2}\Big) - \beta_{x_s} \Omega_0^2 \Big(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2\Big) + \beta_{x_s} \Omega_0 V_0 A_2 (\theta_{y_s}$$

$$+ \alpha_{y_0}) - 3 \beta_{c_s} V_0 \Omega_0 \Big(\theta_{c_{s_R}} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2}\Big)\Big]$$

$$S_{\dot{\beta}_{c_\Delta}} = (h \alpha_{y_0} + x_s)\Big[\beta_{c_s} V_0 \frac{3}{2} A_2 + \beta_{x_s} \Omega_0 \frac{3}{2} C_2\Big]$$

$$S_{\beta_{c_\Delta}} = (h \alpha_{y_0} + x_s)\Big[(\theta_{y_s} + \alpha_{y_0})\Big(V_0^2 \frac{E_2}{2} + \Omega_0^2 \frac{C_2}{2}\Big)$$
$$- 3 V_0 \Omega_0 \Big(\theta_{c_{s_R}} \frac{A_2}{2} - \frac{\theta_d}{R} \frac{C_2}{2}\Big) + \beta_{y_s} V_0^2 E_2$$
$$+ V_0 v \frac{3}{2} E_2 - \beta_{y_s} \Omega_0^2 \frac{C_2}{2}\Big]$$

$$S_{\dot{\beta}_{x_\Delta}} = (h \alpha_{y_0} + x_s)\Big[\beta_{c_s} \Omega_0 \frac{C_2}{2} + \beta_{x_s} V_0 \frac{5}{8} A_2\Big]$$

$$S_{\beta_{x_\Delta}} = (h \alpha_{y_0} + x_s)\Big[- V_0^2 \Big(\theta_{c_{s_R}} \frac{E_2}{2} - \frac{\theta_d}{R} \frac{A_2}{2}\Big) - \Omega_0^2 \Big(\theta_{c_{s_R}} C_2$$
$$- \frac{\theta_d}{R} B_2\Big) + V_0 \Omega_0 A_2 (\theta_{y_s} + \alpha_{y_0}) + \beta_{y_s} V_0 \Omega_0 \frac{A_2}{2}$$
$$+ v \Omega_0 \frac{3}{2} A_2\Big]$$

$$S_{\dot{\beta}_{y_\Delta}} = (h \alpha_{y_0} + x_s)\Big[\beta_{y_s} V_0 \frac{7}{8} A_2 - \Omega_0 \Big(\theta_{c_{s_R}} \frac{C_2}{2} - \frac{\theta_d}{R} \frac{B_2}{2}\Big)$$
$$+ v A_2 + V_0 \frac{A_2}{8} (\theta_{y_s} + \alpha_{y_0})\Big]$$

$$S_{\beta_{y_\Delta}} = (h \alpha_{y_0} + x_s)\Big[\beta_{c_s} V_0^2 E_2 - \beta_{c_s} \Omega_0^2 \frac{C_2}{2}$$
$$+ \beta_{x_s} V_0 \Omega_0 \frac{A_2}{2}\Big]$$

$$S_{\dot{x}} = S_{\dot{\alpha}_{y_\Delta}} / h - l_t V_0 E_4 \theta_{0_t}$$

$$S_{\ddot{y}} = S_{\ddot{\alpha}_{x_\Delta}} / h$$

$$S_{\dot{y}} = S_{\dot{\alpha}_{x_\Delta}}/h + l_t \Omega_{o_t} A_4$$

$$S_{\dot{z}} = -(h\alpha_{y_o} + x_s)\left[\beta_{c_s} V_o \tfrac{3}{2}E_2 + \beta_{x_s}\Omega_o \tfrac{3}{2} A_2\right]$$

$$\begin{aligned} S_{\Omega_\Delta} = {} & (h\alpha_{y_o} + x_s)\Big[(\theta_{y_s} + \alpha_{y_o})(\beta_{c_s}\Omega_o C_2 + \beta_{x_s} V_o A_2) \\ & -3\beta_{c_s} V_o \left(\theta_{c_{s_R}} \tfrac{A_2}{2} - \tfrac{\theta_d}{R}\tfrac{C_2}{2}\right) - 2\beta_{x_s}\Omega_o \Big(\theta_{c_{s_R}} C_2 \\ & - \tfrac{\theta_d}{R} B_2\Big) - \beta_{c_s}\beta_{y_s}\Omega_o C_2 + \beta_{x_s}\left(v \tfrac{3}{2} A_2 + \beta_{y_s} V_o \tfrac{A_2}{2}\right)\Big] \\ & - l_t \frac{\Omega_o}{\Omega_{o_t}} \left(\theta_{o_t} 2\Omega_{o_t} C_4 - v_t A_4\right) \end{aligned}$$

$$\begin{aligned} S_{\theta_{c_\Delta}} = {} & (h\alpha_{y_o} + x_s)\Big[\beta_{x_s}\left(V_o^2 \tfrac{E_2}{2} + \Omega_o^2 C_2\right) \\ & - 3\beta_{c_s} V_o \Omega_o \tfrac{A_2}{2}\Big] \end{aligned}$$

$$\begin{aligned} S_{\theta_{y_\Delta}} = {} & -(h\alpha_{y_o} + x_s)\Big[\beta_{c_s}\left(V_o^2 \tfrac{E_2}{2} + \Omega_o^2 \tfrac{C_2}{2}\right) \\ & + \beta_{x_s} V_o \Omega_o A_2\Big] \end{aligned}$$

$$S_{\theta_{x_\Delta}} = (h\alpha_{y_o} + x_s)\left[\beta_{y_s} V_o \Omega_o \tfrac{A_2}{2} - \Omega_o v \tfrac{A_2}{2}\right]$$

$$S_{\theta_{\Delta_t}} = l_t\left(\Omega_{o_t}^2 C_4 + V_o^2 E_4/2\right)$$

SECTION V

EQUATIONS FOR AERODYNAMIC TORQUE

5.1 MAIN ROTOR TORQUE

The torque force acting on the blade element is given by (see Figure 5):

$$dF_{Q_a} = dD + \phi\, dL \tag{76}$$

Thus, the elemental torque is

$$\begin{aligned} dQ_a &= r\, dF_{Q_a} \\ &= r\left[\rho c \frac{U_T^2}{2} C_{D_0}\, dr + \rho \frac{ca}{2}\left(\theta U_P U_T - U_P^2\right) dr\right] \end{aligned} \tag{77}$$

Using equations (21), (23), and (24), and integrating over the blade span (neglecting the hinge offset), and summing for b blades,

$$\begin{aligned} Q_a = {} & (\dot{x} + h\dot{\alpha}_{y_\Delta})\Big[V_0 A_3 - (\theta_{y_s} + \alpha_{y_0})\left(\beta_{y_s} V_0 \frac{A_2}{4} + v\frac{A_2}{2}\right) \\ & - \beta_{c_s}^2 V_0 A_2 - \beta_{c_s}\beta_{x_s}\Omega_0 C_2 - V_0 \frac{A_2}{4}\left(3\beta_{y_s}^2 + \beta_{x_s}^2\right) \\ & - \beta_{y_s} v A_2\Big] + (\dot{y} + h\dot{\alpha}_{x_\Delta})\Big[(\theta_{y_s} + \alpha_{y_0})\Big(\beta_{x_s} V_0 \frac{A_2}{4} \\ & + \Omega_0 \beta_{c_s}\frac{C_2}{2}\Big) - \beta_{c_s}\beta_{y_s}\Omega_0 C_2 + \beta_{x_s}\beta_{y_s} V_0 \frac{A_2}{2} + \beta_{x_s} v A_2\Big] \\ & + \theta_{y_\Delta}\left[-V_0^2 \beta_{y_s}\frac{A_2}{8} - v V_0 \frac{A_2}{2} + \beta_{y_s}\Omega_0^2 \frac{B_2}{2}\right] + \alpha_{y_\Delta}\Big[-V_0^2 \beta_{y_s}\frac{A_2}{8} \\ & - v V_0 \frac{A_2}{2} + \beta_{y_s}\Omega_0^2 \frac{B_2}{2}\Big] + \dot{\beta}_{x_\Delta}\Big[V_0\left(\theta_{c_{s_R}}\frac{C_2}{2} - \frac{\theta_1}{R}\frac{B_2}{2}\right) \\ & - \Omega_0 \frac{B_2}{2}(\theta_{y_s} + \alpha_{y_0}) + \beta_{y_s}\Omega_0 B_2\Big] + \beta_{x_\Delta}\Big[-V_0^2 \beta_{x_s}\frac{A_2}{4} \\ & - \Omega_0 V_0 \beta_{0_s} C_2 - \Omega_0^2 \beta_{x_s} B_2\Big] + \dot{\beta}_{y_\Delta}\left[-V_0 \beta_{c_s} C_2 - \beta_{x_s}\Omega_0 B_2\right] \\ & + \beta_{y_\Delta}\Big[(\theta_{y_s} + \alpha_{y_0})\left(\Omega_0^2 \frac{B_2}{2} - V_0^2 \frac{A_2}{8}\right) - V_0^2 \beta_{y_s}\frac{3A_2}{4} - V_0 v A_2 \end{aligned}$$

$$
\begin{aligned}
&-\Omega_0^2 \beta_{y_s} B_2] + \dot{\beta}_{c_\Delta}[\Omega_0(\theta_{c_{s_R}} B_2 - \frac{\theta_d}{R} D_2) - V_0 \frac{C_2}{2}(\theta_{y_s} + \alpha_{y_0}) \\
&- V_0 \beta_{y_s} C_2 - 2 v C_2] + \beta_{c_\Delta}[-V_0^2 \beta_{c_s} A_2 - V_0 \beta_{x_s} \Omega_0 C_2] \\
&+ \theta_{c_\Delta}[\Omega_0 v C_2] + \dot{z}[-\Omega_0(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2) + V_0 \frac{A_2}{2}(\theta_{y_s} \\
&+ \alpha_{y_0}) + V_0 \beta_{y_s} A_2 + 2 v A_2] + \Omega_\Delta[2\Omega_0 B_3 + v(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2) \\
&+ \beta_{y_s} \Omega_0 B_2 (\theta_{y_s} + \alpha_{y_0}) - \beta_{x_s} \beta_{c_s} V_0 C_2 - \Omega_0 B_2 (\beta_{x_s}^2 + \beta_{y_s}^2)] \\
&+ \theta_{x_\Delta}[V_0^2 \beta_{x_s} \frac{A_2}{8} + V_0 \Omega_0 \beta_{c_s} \frac{C_2}{2} + \Omega_0^2 \beta_{x_s} \frac{B_2}{2}] \\
&+ \alpha_{x_\Delta}[-V_0^2 \beta_{x_s} \frac{A_2}{8} - V_0 \Omega_0 \beta_{c_s} \frac{C_2}{2} - \Omega_0^2 \beta_{x_s} \frac{B_2}{2}]
\end{aligned}
\tag{78}
$$

5.2 TAIL ROTOR TORQUE

Similarly to the preceding development, the tail rotor torque is given by

$$dQ_t = r_t(\phi_t \, dL_t + dD_t) \tag{79}$$

$$= \frac{\rho c_t r_t}{2}[a_t(\theta_t U_{T_t} U_{P_t} - U_{P_t}^2) + C_{D_{0_t}} U_{T_t}^2] \tag{80}$$

Using equations (42), (43), (44) and (45), integrating, for blades,

$$
\begin{aligned}
Q_t = &(\theta_{0_t} + \theta_{\Delta_t})[\Omega_{0_t} C_4 (v_t - l_t \dot{\alpha}_{z_\Delta} + \dot{y} \\
&- V_0 \alpha_z) + \Omega_{\Delta_t} v_t C_4] - v_t^2 A_4 \\
&- 2 v_t A_4 (-l_t \dot{\alpha}_z + \dot{y} - V_0 \alpha_z) + \Omega_{0_t}^2 B_5 \\
&+ 2\Omega_{0_t} \Omega_{\Delta_t} B_5 + V_0^2 \frac{A_5}{2} + V_0 \dot{x} A_5
\end{aligned}
\tag{81}
$$

SECTION VI

REFERENCES

1. **Warsett, P.:** "Equations of Motion for a Nominally-Hovering Helicopter with Rotor RPM Degree of Freedom", M-H Aero Report AD 5143-TR7, 19 June 1953.

2. **Tamura, J.:** "Control of Altitude and Rotor RPM of the Single-Rotor Helicopter with Reciprocating Engine in Hovering Flight", M-H Aero Report AD 5143-TR10, 15 February 1955.

3. **Mazur, J.:** "An Analytical Method for the Determination of the Basic Longitudinal Stability Characteristics of a Conventional Single-Lifting-Rotor Helicopter", Master's Thesis, University of Maryland, July 1952.

APPENDIX I

THE STEADY-STATE EQUATIONS

The development of the equations of motion in cruising flight as described in the present report included an underlying consideration of both steady-state and transient terms. In the body of this report, the final equations deleted pure steady-state terms, since in any one equation these terms must collectively vanish. However, for purposes of calculating the transient equation coefficients (stability derivatives), the numerical value of many steady-state terms must be determined. This in turn is accomplished by writing and solving the equations of motion under steady-state conditions, that is, the equations having only steady-state terms.

I.1 THE BLADE EQUATIONS

Based on equations (27), (18), and (26), the steady-state equations are as follows:

Unity Terms:

$$\begin{aligned} 0 = {} & V_0^2\left(\theta_{cs_R}\frac{A_1}{2} - \frac{\theta_d}{R}\frac{C_1}{2}\right) + \Omega_0^2\left(\theta_{cs_R}B_1 \right. \\ & \left. - \frac{\theta_d}{R}D_1 - I_b\beta_{cs}\right) - \Omega_0 V_0 C_1\left(\theta_{y_s} + \alpha_{y_0}\right) \\ & - \Omega_0 v C_1 - g J_b \end{aligned} \tag{I-1}$$

Sine Terms:

$$\begin{aligned} 0 = {} & V_0^2\left[-\left(\theta_{y_s} + \alpha_{y_0}\right)\frac{3}{4}A_1 - \beta_{y_s}\frac{A_1}{4}\right] \\ & + \Omega_0^2\left[-\left(\theta_{y_s} + \alpha_{y_0}\right)B_1 + \beta_{y_s}B_1\right] \\ & + \Omega_0 V_0\left[2\theta_{cs_R}C_1 - 2\frac{\theta_d}{R}B_1\right] \\ & - V_0 v A_1 \end{aligned} \tag{I-2}$$

Cosine Terms:

$$0 = -V_0^2 \beta_{x_s} \frac{A_1}{4} - \Omega_0^2 \beta_{x_s} B_1 - \Omega_0 V_0 \beta_{c_s} C_1 \tag{I-3}$$

I.2 THE HELICOPTER FORCE EQUATIONS

In place of the equations (31), (32), (33) for transient motion, the corresponding steady-state equations are

$$F_{x_{R_{s.s.}}} - \text{Fuselage Drag} = 0 \tag{I-4}$$

$$F_{Y_{R_{s.s.}}} + T_{t_{s.s.}} = 0 \tag{I-5}$$

$$F_{Z_{R_{s.s.}}} + \text{Fuselage Weight} = 0 \tag{I-6}$$

From equation (37), the steady-state rotor-force in the X direction is substituted into equation (I-4) above to give

$$\begin{aligned} D_F = {} & \Omega_0^2 \left[\beta_{y_s}\left(\theta_{c_{s_R}} C_2 - \frac{\theta_d}{R} B_2\right) - \beta_{c_s}\beta_{x_s}\frac{C_2}{2}\right] \\ & + \Omega_0 V_0\left[-\beta_{y_s}(\theta_{y_s} + \alpha_{y_0})\frac{A_2}{2} - (\beta_{c_s}^2 + \beta_{y_s}^2)\frac{A_2}{4} \right. \\ & \left. - A_3\right] + \Omega_0\left[-\beta_{y_s} v \frac{3}{2} A_2 + v \frac{A_2}{2}(\theta_{y_s} \right. \\ & \left. + \alpha_{y_0})\right] - V_0\left[v\left(\theta_{c_{s_R}}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right)\right] \end{aligned} \tag{I-7}$$

For equation (I-6), with the positive Z direction being downward, referring to equation (39), there is obtained

$$\begin{aligned} -W_h = {} & -V_0^2\left(\theta_{c_{s_R}}\frac{E_2}{2} - \frac{\theta_d}{R}\frac{A_2}{2}\right) - \Omega_0^2\left(\theta_{c_{s_R}} C_2 \right. \\ & \left. - \frac{\theta_d}{R} B_2\right) + \Omega_0 V_0(\theta_{y_s} + \alpha_{y_0}) A_2 \\ & + \Omega_0 v A_2 + b W_b \end{aligned} \tag{I-8}$$

I.3 THE HELICOPTER MOMENT EQUATIONS

In steady-state flight, the three equations corresponding to equations (70), (72), and (74) are

$$F_{x_{R_{s.s.}}} h - F_{x_{R_{s.s.}}} x_s \alpha_{y_0} + F_{z_{R_{s.s.}}} h \alpha_{y_0} + F_{z_{R_{s.s.}}} x_s + M_{y_{R_{s.s.}}} + M_{y_{F_{s.s.}}} + M_{y_{T_{s.s.}}} = 0 \qquad \text{(I-9)}$$

$$F_{y_{R_{s.s.}}} h + M_{x_{R_{s.s.}}} + T_{t_{s.s.}} h_t = 0 \qquad \text{(I-10)}$$

$$Q_{e_{s.s.}} + F_{y_{R_{s.s.}}} h \alpha_{y_0} - T_{t_{s.s.}} l_t = 0 \qquad \text{(I-11)}$$

To expand these equations, it is required to obtain several steady-state terms as follows. The rotor moment $M_{y_{R\,s.s.}}$ was obtained identically as M_y in equation (56). In other words, beginning with equation (51), and retaining only steady-state terms, there is obtained

$$\begin{aligned} M_{y_{R_{s.s.}}} = {} & e\,\alpha_{y_0} \Big\{ \Omega_0^2 \Big[-\frac{L_y}{2} + \beta_{c_s}\Big(\theta_{c_{s_R}} \frac{C_2}{2} - \frac{\theta_d}{R}\frac{B_2}{2}\Big) \\ & - \beta_{y_s}\beta_{x_s}\frac{C_2}{2}\Big] + \Omega_0 V_0 \Big[-(\theta_{y_s} + \alpha_{y_0})\beta_{c_s}\frac{A_2}{8} \\ & - \beta_{c_s}\beta_{y_s}\frac{7}{8}A_2 \Big] + \Omega_0\Big[-\beta_{c_s} v \frac{A_2}{2}\Big] \\ & + V_0\Big[\beta_{x_s} v \frac{\epsilon_2}{8}\Big]\Big\} + e\Big\{ -V_0^2\Big[\beta_{x_s}\frac{\epsilon_2}{8}\Big] \\ & + \Omega_0^2\Big[\beta_{y_s}\frac{L_y}{2} - \beta_{x_s}\frac{C_2}{2}\Big] - V_0\Omega_0\Big(\beta_{c_s}\frac{A_2}{2}\Big)\Big\} \end{aligned} \qquad \text{(I-12)}$$

The steady-state fuselage moment is obtained from equation (58) using the steady-state moment coefficient. The steady-state tail moment is obtained from equation (61) using the steady-state inclination angle α_{y_0} .

APPENDIX II

SPECIFIED PARAMETERS FOR HRS-3 HELICOPTER IN CRUISING FLIGHT CONDITION

II.1 The sources of data as well as much of the data used here can be found in Appendix II of Reference (2)

Gross Weight = 7450 pounds

Blade-less Helicopter Weight = 7035 pounds

Blade Weight = 138.3 pounds

h = 6.9 feet

b = 3

c = 1.37 feet

e = 0.75 feet

I_b = 925 slug ft^2 (blade moment of inertia about flapping hinge)

a = a_t = 5.73/radian

C_{D_0} = 0.012 at operating C_L

Ω_0 = 22.2 rad/sec.

R_t = 4.333 feet

b_t = 2

Tail rotor blade root chord = 1.25 feet

Tail rotor blade tip chord = 0.5 feet

Gear ratio, engine to main rotor = 11.315 : 1

Gear ratio, engine to tail rotor = 1.62 : 1

h_t = 5.58 feet

l_t = 31.33 feet

m = 0.167 slugs/ft.

Main rotor blades have a linear twist of 8° between r = 0 and r = R. (θ_d = 0.1396 rad.)

Airfoil sections begin at r = ϵ = e + 40.25" = 4.1 ft.

R = 26.5 feet

g = 32.2 fps^2

ρ = 0.002378 slugs/ft^3

A_T = 6.7 sq. ft.

V_0 = 90 knots

B = 0.97

l_T = 28 ft.

I_y = 10,000 slug ft^2

I_x = 2200 slug ft^2

I_z = 9181 slug ft^2

I_{E-R} = 3213.6 slug ft^2

<u>II.2</u> The brake horsepower required for the assumed flight condition was found from

(a) HRS-3 Handbook AN 01-230HJA-2
(b) Figure 1 of M-H Aero Report AD 5143-TR9

to be 525.

It was assumed that the power delivered to the main rotor was

$$0.85 \times 525 = \underline{446 \text{ HP}}$$

This power was used for (a) main rotor induced losses, (b) main rotor profile losses, and (c) helicopter parasite losses. These were determined as follows: From Rotary Wing Aircraft Handbooks and History (Vol. 6) - Aerodynamics and Performance of Helicopters, by Klemin and Sikorsky, page 51,

$$C_T = \frac{W}{\pi R^2 \rho (\Omega_o R)^2} = 0.00413$$

$$\sigma = bc/\pi R = 0.0494$$

$$2\, C_T/\sigma = 0.167$$

$$HP_i \cdot \mu = C_T^2 \rho \pi R^2 (\Omega_o R)^3 / (2B^2 550)$$

$$= 17.6$$

$$\mu = V_o/\Omega_o R = 0.258$$

Thus $HP_i = \underline{68}$

$$C_{L(m)} = \frac{2\,C_T}{\sigma} \frac{1}{\frac{1}{3} + \frac{\mu^2}{2}}$$ (see page 54 of aforementioned handbook)

$C_{L(m)}$ = 0.456

F_{29} = 1.22 (")

C_{Q_0} = 0.0000828 (")

Thus

$$HP_o = C_{Q_o} \rho \pi R^2 (\Omega_o R)^3 / 550$$

HP_o = $\underline{161}$

C_{D_f} = 0.013 (estimated, assumed)

$$D_f = \tfrac{1}{2} \rho V_o^2 C_{D_f} \pi R^2 = \underline{788}\,\#$$

$$HP_{Par.} = \frac{D_f \cdot V_o}{550} = \underline{217}$$

$$BHP = 68 + 161 + 217 = \underline{446}$$

<u>II.3</u> The induced velocity was found from

$$v = T / \left[2\pi (BR)^2 \sqrt{V_o^2 + v^2}\, \rho\right]$$

(see page 37 of aforementioned handbook)

$$T = \sqrt{D_f^2 + W^2}$$

T = 7492 pounds

v = 5 fps.

II.4 The tail rotor induced velocity was found from

$$v_t = \frac{T_t}{2\pi R_t^2 \rho \sqrt{V_0^2 + v_t^2}}$$

T_t = 353 pounds (see IV.1)

Thus, v_t = 8.4 fps.

II.5 Using the steady-state terms in equation (47), with

Ω_{0_t} = 155.14 rad/sec,

the tail rotor pitch angle is found to be

θ_{0_t} = 0.071 rad.

II.6 Taking the tail rotor power to be 3% of the engine brake horsepower,

Q_t = 55.8 ft. lb.

On this basis, the steady-state terms in equation (81) lead to

$C_{D_{0_t}}$ = 0.0113

II.7 The fuselage longitudinal moment characteristics were taken largely from Reference (3), in which wind tunnel data obtained on a 1/10-scale model of the HRS-3 helicopter fuselage were reproduced. The following expression represents a good approximation of these moment characteristic data:

$$C_{m_f} = -0.0002 + 0.01185\alpha_{y_0} - 0.0361\alpha_{y_0}^2 \qquad \text{(II-1)}$$

II.8 The fixed tail surface pitching moment was also found with the help of Reference (3). The inverted V-type tail surfaces of the HRS-3 are represented by a straight rectangular

surface having identical projected span and chord. The aspect ratio of this "equivalent" tail was 3.18 and its area was S_T = 5.65 sq.ft. The value of a_T taken in Reference (3) was 3.56. The effective longitudinal stabilizer pitch setting (neutral position)- see Figure 8a - was given in Reference (3) as i_T = -0.1757 radians. Thus, the angle of attack of the tail (with positive lift directed downward) is

$$\alpha_{T_0} = i_T + \alpha_{y_0} + v/V_0$$

$$= -0.1757 + \alpha_{y_0} + 0.0329$$

$$= -0.1428 + \alpha_{y_0} \qquad \text{(II-2)}$$

APPENDIX III

EVALUATION OF INTEGRAL SYMBOLS

$$A_1 = \frac{\rho c a}{2} \int_{\epsilon}^{BR} r\,dr = 3.005$$

$$A_2 = b A_1 = 9.015$$

$$A_3 = \frac{\rho c b C_{D_0}}{2} \int_{e}^{R} r\,dr = 0.0206$$

$$A_4 = \frac{\rho b_t a_t}{2} \int_{0}^{R_t} c_t r_t\,dr_t = 0.096$$

$$A_5 = A_4\, C_{D_{0t}} / a_t = 0.000189$$

$$B_1 = \frac{\rho c a}{2} \int_{\epsilon}^{BR} r^3\,dr = 1018$$

$$B_2 = B_1 \times b = 3054$$

$$B_3 = C_{D_0} \frac{\rho c b}{2} \int_{e}^{R} r^3\,dr = 7.23$$

$$B_4 = \frac{\rho b_t a_t}{2} \int_{0}^{R_t} c_t r_t^3\,dr = 0.7793$$

$$B_5 = B_4\, C_{D_{0t}} / a_t = 0.00154$$

$$C_1 = \frac{\rho c a}{2} \int_{\epsilon}^{BR} r^2\,dr = 52.63$$

$$C_2 = C_1 \times b = 157.9$$

$$C_3 = C_{D_0} \frac{\rho c b}{2} \int_{e}^{R} r^2\,dr = 0.364$$

$$C_4 = \frac{\rho b_t a_t}{2} \int_{0}^{R_t} c_t r_t^2\,dr_t = 0.254$$

$$C_5 = C_4 \times C_{D_{0t}}/a_t = 0.0005$$

$$D_1 = \frac{\rho c a}{2}\int_{\epsilon}^{BR} r^4\,dr = 20948$$

$$D_2 = D_1 \times b = 62844$$

$$D_3 = \frac{C_{D_0}\rho c b}{2}\int_{e}^{R} r^4\,dr = 153.4$$

$$E_1 = \frac{\rho c a}{2}\int_{\epsilon}^{BR} dr = 0.2016$$

$$E_2 = E_1 \times b = 0.6048$$

$$E_3 = C_{D_0}\frac{\rho c b}{2}\int_{e}^{R} dr = 0.0015$$

$$E_4 = \rho\frac{b_t}{2}a_t\int_{0}^{R_t} c_t\,dr_t = 0.0517$$

$$E_5 = E_4 \times C_{D_{0t}}/a_t = 0.0001$$

$$J_b = m\int_{e}^{R} r\,dr = 58.59$$

$$L_4 = b\int_{e}^{R} m r\,dr = 175.8$$

APPENDIX IV

SOLUTION OF STEADY-STATE EQUATIONS

IV.1 Substituting from equation (I-5), the torque equation (I-11) becomes

$$Q_{e_{s.s.}} = T_{t_{s.s.}} (l_t - h\,\alpha_{y_0})$$

With only very small error, this can be written

$$T_{t_{s.s.}} = \frac{Q_{e_{s.s.}}}{l_t} = \frac{446 \times 550}{22.2 \times 31.33} = \underline{353^{\#}} \qquad \text{(IV-1)}$$

IV.2 Substituting numerical values from Appendices II and III, equations I-1, I-2, I-3 become

$$536425\,\theta_{c_{s_R}} - 177595\,\theta_{y_s} - 177595\,\alpha_{y_0} - 455877\,\beta_{c_s} - 65318 = 0 \qquad \text{(IV-2)}$$

$$355189\,\theta_{c_{s_R}} - 553782\,\theta_{y_s} - 553782\,\alpha_{y_0} + 484354\,\beta_{y_s} - 38476 = 0 \qquad \text{(IV-3)}$$

$$-519068\,\beta_{x_s} - 177595\,\beta_{c_s} = 0 \qquad \text{(IV-4)}$$

IV.3 Substituting numerical values from Appendices II and III, equations I-7, I-8 become

$$77819\,\theta_{c_{s_R}}\beta_{y_s} - 230\,\theta_{c_{s_R}} - 15210\,\theta_{y_s}\beta_{y_s} + 500\,\theta_{y_s} - 15210\,\alpha_{y_0}\beta_{y_s} + 500\,\alpha_{y_0} - 9430\,\beta_{y_s} - 15210\,\beta_{y_s}^2 - 38910\,\beta_{x_s}\beta_{c_s} - 15210\,\beta_{c_s}^2 - 840 = 0 \qquad \text{(IV-5)}$$

$$-84806\,\theta_{cs_R} + 30420\,\theta_{y_s} + 30420\,\alpha_{y_0} + 16929 = 0 \qquad \text{(IV-6)}$$

IV.4 The fuselage moment, as given by equations (58) and (II-1) is

$$M_{y_{f_{s.s.}}} = -321.2 + 20878.6\,\alpha_{y_0} - 30514.8\,\alpha_{y_0}^2 \qquad \text{(IV-7)}$$

The tail moment, as given by equation (61) and the material in Appendix II.8, is

$$M_{y_{T_{s.s.}}} = 2211.8 - 15488\,\alpha_{y_0} \qquad \text{(IV-8)}$$

IV.5 The steady-state pitching moment equation (I-9) can now be written with numerical substitution for the parameters. It was decided that here, too, the C.G. offset x_s could be neglected with only small error. Thus,

$$\begin{aligned}
&\theta_{cs_R}(29182\,\beta_{c_s}\alpha_{y_0} + 536951\,\beta_{y_s} - 1587 - 585161\,\alpha_{y_0}) \\
&+ \theta_{y_s}(-2852\,\beta_{c_s}\alpha_{y_0} - 104949\,\beta_{y_s} + 3450 + 209898\,\alpha_{y_0}) \\
&+ \alpha_{y_0}(-3348\,\beta_{c_s} - 29182\,\beta_{x_s}\beta_{y_s} - 2852\,\beta_{c_s}\alpha_{y_0} \\
&\quad - 19963\,\beta_{c_s}\beta_{y_s} + 43\,\beta_{x_s} - 104949\,\beta_{y_s} + 44616 \\
&\quad + 179383\,\alpha_{y_0}) - 30492\,\beta_{x_s} - 32577\,\beta_{y_s} \\
&\quad - 11408\,\beta_{c_s} + 1532 - 104949\,\beta_{y_s}^2 \\
&\quad - 268479\,\beta_{x_s}\beta_{c_s} - 104949\,\beta_{c_s}^2 = 0 \qquad \text{(IV-9)}
\end{aligned}$$

IV.6 The six independent steady-state equations (IV-2), (IV-3), (IV-4), (IV-5), (IV-6), and (IV-9) contain the six basic steady-state parameters. Solving these equations simultaneously gave

$$\alpha_{y_o} = 0.138 \text{ radians}$$

$$\beta_{y_s} = 0.117 \text{ radians}$$

$$\beta_{x_s} = -0.0337 \text{ radians}$$

$$\beta_{c_s} = 0.0984 \text{ radians}$$

$$\theta_{y_s} = 0.0722 \text{ radians}$$

$$\theta_{c_{s_R}} = 0.275 \text{ radians}$$

The remaining steady-state equations (I-5), (I-10), (I-11) have been essentially used in the various calculations in Appendix II and IV.1. Note that in the present work it is assumed that the built-in lateral component of rotor thrust is such that $\theta_{x_{s.s.}}$ can be taken as zero.

APPENDIX V

EQUATIONS OF MOTION WITH NUMERICAL COEFFICIENTS

Using the numerical data in the preceding sections of the Appendix, the equation coefficients and stability derivatives can be evaluated. On this basis, the equations in the text of the report were transformed into the following:

V.1 Equation (28)

$$\begin{aligned}
&\ddot{\beta}_{c_\Delta} + 24.4\,\dot{\beta}_{c_\Delta} + 493\,\beta_{c_\Delta} + 0.0256\,\ddot{\alpha}_{y_\Delta} \\
&+ 1.21\,\dot{\alpha}_{y_\Delta} + 192\,\alpha_{y_\Delta} + 0.0074\,\ddot{\alpha}_{x_\Delta} + 4.32\,\dot{\beta}_{x_\Delta} \\
&+ 0.0037\,\ddot{x} + 0.174\,\dot{x} + 0.001\,\ddot{y} - 0.0633\,\ddot{z} \\
&- 1.26\,\dot{z} - 1.69\,\Omega_\Delta - 580\,\Theta_{c_\Delta} \\
&- 192\,\Theta_{y_\Delta} = 0
\end{aligned} \tag{V-1}$$

V.2 Equation (29)

$$\begin{aligned}
&\ddot{\beta}_{y_\Delta} + 24.4\,\dot{\beta}_{y_\Delta} + 44.4\,\dot{\beta}_{x_\Delta} + 561\,\beta_{x_\Delta} \\
&+ 192\,\beta_{c_\Delta} + 0.043\,\ddot{\alpha}_{y_\Delta} + 0.8\,\dot{\alpha}_{y_\Delta} \\
&- 2.34\,\dot{\alpha}_{x_\Delta} + 561\,\alpha_{x_\Delta} + 0.006\,\ddot{x} \\
&+ 0.116\,\dot{x} - 0.34\,\dot{y} - 0.034\,\dot{\Omega}_\Delta \\
&- 0.8\,\Omega_\Delta + 0.5\,\Omega_\Delta/s \\
&- 561\,\Theta_{x_\Delta} = 0
\end{aligned} \tag{V-2}$$

V.3 Equation (30)

$$
\begin{aligned}
&\ddot{\beta}_{x_\Delta} + 24.4\,\dot{\beta}_{x_\Delta} - 44.4\,\dot{\beta}_{y_\Delta} - 524\,\beta_{y_\Delta} \\
&+ 8.65\,\dot{\beta}_{c_\Delta} - 1.62\,\dot{\alpha}_{y_\Delta} + 599\,\alpha_{y_\Delta} - 0.043\,\ddot{\alpha}_{x_\Delta} \\
&- 0.801\,\dot{\alpha}_{x_\Delta} - 0.237\,\dot{x} - 0.006\,\ddot{y} \\
&- 0.116\,\dot{y} - 0.49\,\ddot{z} - 0.117\,\dot{\Omega}_\Delta + 1.55\,\Omega_\Delta \\
&+ 0.013\,\frac{\Omega_\Delta}{s} - 599\,\theta_{y_\Delta} - 384\,\theta_{c_\Delta} = 0
\end{aligned}
\qquad \text{(V-3)}
$$

V.4 Equation (48)

$$
\begin{aligned}
&\ddot{x} + \left(0.024 + \begin{smallmatrix}0.0519\\ \text{or } *\\ 0\end{smallmatrix}\right)\dot{x} + 0.385\,\ddot{\alpha}_{y_\Delta} \\
&+ 166\,\dot{\alpha}_{y_\Delta} + 5.54\,\alpha_{y_\Delta} - 0.09\,\dot{\alpha}_{x_\Delta} + 16.5\,\alpha_{x_\Delta} \\
&+ 0.772\,\dot{\beta}_{y_\Delta} - 22.6\,\beta_{y_\Delta} + 0.562\,\dot{\beta}_{x_\Delta} \\
&+ 16.5\,\beta_{x_\Delta} + 1.62\,\dot{\beta}_{c_\Delta} + 7.33\,\beta_{c_\Delta} \\
&- 0.013\,\dot{y} - 0.1\,\dot{z} - 0.5\,\Omega_\Delta \\
&- 16.5\,\theta_{x_\Delta} - 5.54\,\theta_{y_\Delta} - 38.4\,\theta_{c_\Delta} \\
&= 0
\end{aligned}
\qquad \text{(V-4)}
$$

V.5 Equation (49)

$$
\ddot{y} + 0.076\,\dot{y} + 0.385\,\ddot{\alpha}_{x_\Delta} + 0.0772\,\dot{\alpha}_{x_\Delta}
$$

---

* If fuselage effects are neglected.

$$
\begin{aligned}
&- 9.87\,\alpha_{x_\Delta} + 0.0642\,\dot{\alpha}_{y_\Delta} - 15.1\,\alpha_{y_\Delta} \\
&+ 0.356\,\dot{\beta}_{y_\Delta} + 12.8\,\beta_{y_\Delta} - 0.622\,\dot{\beta}_{x_\Delta} \\
&+ 22.2\,\beta_{x_\Delta} - 0.107\,\dot{\beta}_{c_\Delta} + 4.10\,\beta_{c_\Delta} \\
&- 2.02\,\dot{\alpha}_{z_\Delta} - 9.80\,\alpha_{z_\Delta} + 0.00684\,\dot{x} \\
&+ 0.0149\,\dot{z} - 0.238\,\Omega_\Delta + 15.1\,\Theta_{y_\Delta} \\
&+ 9.87\,\Theta_{x_\Delta} + 7.12\,\Theta_{c_\Delta} - 29\,\Theta_{\Delta_t} = 0
\end{aligned}
\qquad \text{(V-5)}
$$

V.6 Equation (50)

$$
\begin{aligned}
&\ddot{z} + 0.866\,\dot{z} - 0.716\,\dot{\alpha}_{y_\Delta} \\
&- 132\,\alpha_{y_\Delta} - 0.761\,\ddot{\beta}_{c_\Delta} - 15.2\,\dot{\beta}_{c_\Delta} \\
&- 2.97\,\beta_{x_\Delta} - 0.104\,\dot{x} + 3.80\,\Omega_\Delta \\
&+ 367\,\Theta_{c_\Delta} + 132\,\Theta_{y_\Delta} = 0
\end{aligned}
\qquad \text{(V-6)}
$$

V.7 Equation (71)

$$
\begin{aligned}
&\ddot{\alpha}_{y_\Delta} + \left(+0.0215 + \begin{smallmatrix}0.573\\ \text{or}\\ 0\end{smallmatrix}\right)\dot{\alpha}_{y_\Delta} - \left(-8.95 + \begin{smallmatrix}1.103\\ \text{or}\\ 0\end{smallmatrix} - \begin{smallmatrix}2.78\\ \text{or}\\ 0\end{smallmatrix}\right)\alpha_{y_\Delta} \\
&- 0.0286\,\dot{\alpha}_{x_\Delta} + 5.72\,\alpha_{x_\Delta} + 0.00659\,\ddot{\beta}_{y_\Delta} \\
&+ 0.257\,\dot{\beta}_{y_\Delta} - 6.84\,\beta_{y_\Delta} + 0.383\,\dot{\beta}_{x_\Delta} + 5.74\,\beta_{x_\Delta} \\
&+ 0.259\,\dot{\beta}_{c_\Delta} + 2.28\,\beta_{c_\Delta} + 0.0089\,\ddot{x} \\
&+ 0.00454\,\dot{x} - 0.00416\,\dot{y} - \left(0.0161 + \begin{smallmatrix}.0183\\ \text{or}\\ 0\end{smallmatrix}\right)\dot{z} \\
&- 0.0002\,\dot{\Omega}_\Delta - 0.0776\,\Omega_\Delta - 5.74\,\Theta_{x_\Delta} \\
&- 0.887\,\Theta_{y_\Delta} - 6.16\,\Theta_{c_\Delta} = 0
\end{aligned}
\qquad \text{(V-7)}
$$

V.8 Equation (72)

$$-1.28\,\ddot{\alpha}_{x_\Delta} - 0.0775\,\dot{\alpha}_{x_\Delta} - 29.6\,\alpha_{x_\Delta} - 0.089\,\dot{\alpha}_{y_\Delta}$$
$$+ 26.2\,\alpha_{y_\Delta} + 0.03\,\ddot{\beta}_{x_\Delta} + 1.05\,\dot{\beta}_{x_\Delta} - 30.9\,\beta_{x_\Delta}$$
$$- 1.59\,\dot{\beta}_{y_\Delta} - 21.8\,\beta_{y_\Delta} + 0.304\,\dot{\beta}_{c_\Delta} - 2.9\,\beta_{c_\Delta}$$
$$- 0.0114\,\dot{x} - 0.0405\,\ddot{y} - 0.049\,\dot{y} - 0.0264\,\dot{z} + 1.18\,\dot{\alpha}_{z_\Delta}$$
$$+ 5.74\,\alpha_{z_\Delta} - 0.00764\,\dot{\Omega}_\Delta + 0.222\,\Omega_\Delta - 7.17\,\theta_{x_\Delta}$$
$$- 26\,\theta_{y_\Delta} - 15.6\,\theta_{c_\Delta} + 17\,\theta_{\Delta t} = 0 \qquad \text{(V-8)}$$

V.9 Equation (74)

Using $$Q_{e_\Delta} = \left(\frac{5492.9 + 13723.4\,S}{S^2 + 3.82\,S + 3.6}\right) T_h - 16.32\,\Omega_\Delta \qquad \text{(V-9)}$$

as given in Reference (2), equation (III-6a), equation (74) here becomes

$$-\ddot{\alpha}_{z_\Delta} - 1.59\,\dot{\alpha}_{z_\Delta} - 7.72\,\alpha_{z_\Delta} - 0.00155\,\dot{\alpha}_{y_\Delta}$$
$$+ 0.362\,\alpha_{y_\Delta} - 0.00922\,\ddot{\alpha}_{x_\Delta} - 0.00186\,\dot{\alpha}_{x_\Delta}$$
$$- 0.359\,\alpha_{x_\Delta} + 0.0149\,\dot{\beta}_{x_\Delta} - 0.537\,\beta_{x_\Delta}$$
$$- 0.00349\,\dot{\beta}_{y_\Delta} - 0.308\,\beta_{y_\Delta} + 0.00259\,\dot{\beta}_{c_\Delta}$$
$$- 0.0962\,\beta_{c_\Delta} - 0.00213\,\dot{x} - 0.00134\,\ddot{y}$$
$$+ 0.0505\,\dot{y} - 0.0004\,\dot{z} - 0.114\,\Omega_\Delta$$
$$+ \left(\frac{0.59879 + 1.4948\,S}{S^2 + 3.82\,S + 3.6}\right) T_h - 0.362\,\theta_{y_\Delta} - 0.236\,\theta_{x_\Delta} \qquad \text{(V-10)}$$
$$- 0.169\,\theta_{c_\Delta} - 22.9\,\theta_{\Delta t} = 0$$

V.10 Torque Equilibrium Equation

The equation for rotor angular velocity, as given in Reference (2), equation III-1, is

$$G\,Q_{e_\Delta} - \left(Q_{a_\Delta} + Q_{t_\Delta}\right) = I_{E-R}\,\dot{\Omega}_\Delta \qquad \text{(V-11)}$$

With the numerical data presented in the foregoing sections, and equations (78), (81), and (V-9), this becomes

$$
\begin{aligned}
& -\dot{\Omega}_\Delta - 0.431\,\Omega_\Delta + 0.0675\,\dot{\alpha}_{y_\Delta} - 25.4\,\alpha_{y_\Delta} \\
& + 0.0229\,\dot{\alpha}_{x_\Delta} - 0.00753\,\alpha_{x_\Delta} - 0.899\,\dot{\beta}_{x_\Delta} \\
& - 0.0151\,\beta_{x_\Delta} + 0.0239\,\dot{\beta}_{y_\Delta} + 15.1\,\beta_{y_\Delta} \\
& - 1.37\,\dot{\beta}_{c_\Delta} + 0.79\,\beta_{c_\Delta} + 0.0098\,\dot{x} \\
& + 0.0038\,\dot{y} - 0.0156\,\dot{\alpha}_{z_\Delta} - 0.0759\,\alpha_{z_\Delta} + 0.0663\,\dot{z} \\
& + \left(\frac{19.34 + 48.32\,S}{S^2 + 3.82\,S + 3.6}\right) Th - 5.45\,\Theta_{c_\Delta} \\
& + 0.00753\,\Theta_{x_\Delta} - 0.00005\,\Theta_{\Delta_t} + 25.4\,\Theta_{y_\Delta} = 0
\end{aligned}
\qquad \text{(V-12)}
$$

APPENDIX VI

SYMBOLS*

B = blade lift tip loss factor

C_{D_f} = fuselage drag coefficient

C_{D_o} = profile drag coefficient of blade element

C_L = lift coefficient of blade element

C_{m_f} = moment coefficient of fuselage

C_R = chord length at root of tail rotor blade

C_T = chord length at tip of tail rotor blade

D_f = fuselage drag force

F_{X_F} = transient fuselage force in X direction

$F_{X_R}, F_{Y_R}, F_{Z_R}$ = rotor force in X, Y, Z direction

G = engine-rotor gear ratio

I_{E-R} = moment of inertia of engine-rotor system

$I_{X,Y,Z}$ = moment of inertia of helicopter about indicated axis (not including main rotor blades)

L_T = fixed tail surface lift force

M_a = blade aerodynamic moment about flapping hinge

M_i = blade inertia moment about flapping hinge

M_w = blade weight moment about flapping hinge

M_y, M_x = moment due to blade hinge offset (see eq. 51,52)

$M_{y_{f,T}}$ = longitudinal moment of fuselage, of tail

$M_{y_{T_\alpha}}$ = (see equation 68)

----

* See also Appendix III.

Q_a = main rotor aerodynamic torque

Q_{e_Δ} = engine torque change

Q_t = tail rotor torque

R = rotor radius

T_t = tail rotor thrust

$U_{P,T,R}$ = blade element velocities (see Figure 5)

V = helicopter forward velocity

W = helicopter weight (without blades)

X, Y, Z = coordinate axes (see Figure 1)

a = blade element lift curve slope

a_n = normal acceleration (see Figure 4)

b = number of blades in rotor

c = blade element chord length

dD, dL = blade element drag and lift forces

dF_{Q_a} = blade element torque force

e = flapping hinge offset distance

g = acceleration of gravity

h = distance from rotor hub to c.g. of blade-less helicopter

h_t = See Figure 10

i_T = See Figure 8

l_T = See Figure 8

l_t = See Figure 11

m_b = mass of blade per foot of span

r = distance from rotor center to blade element

S_T = area of fixed tail surface

v = rotor induced velocity

x, y, z = displacement of helicopter c.g.

x_b, y_b, z_b = coordinate location of blade element

x_s = longitudinal distance between helicopter c.g. and rotor shaft

α_ψ = rotor shaft inclination in the vertical plane containing the blade

$\alpha_{x,y,z}$ = angle of helicopter roll, pitch, yaw

β = blade flapping angle at azimuth ψ

$\beta_c, \beta_x, \beta_y$ = See equation (3)

ϵ = innermost blade radius having airfoil shape

θ = blade element pitch angle

$\theta_{c_{s_R}}$ = steady-state collective pitch angle at main rotor root

θ_d = built-in blade angular twist rate (see eq. 24)

θ_y, θ_x = main rotor cyclic pitch angles (see eq. 24)

ρ = mass density of air

$\bar{\rho}$ = See equation (17)

ϕ = blade element wind angle

ψ = blade azimuth angle

Ω = rotor angular velocity

<u>Subscripts</u>

o, s.s. = steady-state

Δ = change, transient value

t = tail rotor

r = main rotor

T = tail surface

f = fuselage